The Effect of Ordering Policy Based on Extended Time-Delay Feedback Control on the Chaotic Behavior in Supply Chains

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doi: http://dx.doi.org/10.13005/bbra/2191

(Received: 15 June 2015; accepted: 24 July 2015)

A supply chain is a complex nonlinear system involving multiple levels and may have a chaotic behavior. The policy of each level in inventory control, demand forecast, and constraints and uncertainties of demand and supply (or production) significantly affects the complexity of its behavior. This paper compares the performance of new ordering policy based on extended time-delay feedback (ETDF) control with the well-known smooth ordering policy on the chaotic behavior in the supply chain. Exponential smoothing (ES) forecast method is used to predict the demand. The effects of inventory adjustment parameter and supply line adjustment parameter on the behavior of the supply chain are investigated. Finally, two scenarios are designed for analysis the chaotic behavior of the supply chain and in each scenario the maximum Lyapunov exponent is calculated and drawn. Finally, the best scenario for decision-making is obtained.

Key words: Supply chain; Ordering policy; Chaotic behavior; Extended time-delay feedback control.

The chaotic behavior (an unusual behavior of nonlinear dynamics) has been observed in supply chains. Mosekilde and Larsen (1988), Thomsen et al. (1992), Sosnotseva and Mosekilde (1997), and Larsen et al. (1999) have considered a deterministic supply chain and have shown its chaotic behavior. They have classified the behavior of this chain in four groups, namely, stable, periodic, chaotic, and hyperchaotic.

One type of dynamic behavior is caused by marketing and competition activities that create interaction between suppliers and customers. The interaction may generate a chaotic behavior in the supply chain (Jarsulic, 1993; Matsumoto, 2001). The changing of price has a fundamental effect on customers demand. Usually the demand goes down as price increases and vice versa. Wu and Zhang (2007) showed the chaotic behavior of the supply chain by simulating the interaction between customers and suppliers where customers respond to the price discount offer made by the supplier and the supplier adjusts the price according to stock held.

Hwarng and Xie (2008) introduced five main factors that influence the supply chain, namely, demand pattern, ordering policy, demand-information sharing, lead time, and supply chain level. They showed the chaotic behavior of the supply chain and its sensitivity to small changes of inventory control parameters using the beer distribution model (Jarmain, 1963) and Sterman dynamic equations (Sterman, 1989). They also quantified the degree of system chaos using the maximum Lyapunov exponent (LE) across all level of the supply chain.
This paper is concerned with the comparison of the ordering policy based on extended time-delay feedback (ETDF) control (Fradkov and Evans, 2005) to the smooth ordering policy. The particular emphasis of this paper is the impact the two ordering policies have on the chaotic behavior in the supply chain. A general class of multi-level supply chain is provided that has four successive levels based on the beer distribution model. Each level must satisfy demand, control inventory and place an order through interactions with adjacent levels. The exponential smoothing (ES) forecast method is used to forecast demand at all levels.

Two scenarios are designed for examining the chaotic behavior of the supply chain based on the forecast method and two ordering policies. In each scenario, the effects of the inventory adjustment parameter and supply line adjustment parameter on the supply chain behavior are investigated through calculating the maximum LE and then the best scenario is selected.

This paper is organized as follows: Section 2 briefly introduces chaotic systems and the extended time-delay feedback (ETDF) control. The section 3 describes a multi-level model of the supply chain and defines its dynamic equations. In addition, the demand forecasting method and ordering policies are assessed. In section 4, a four-level supply chain is simulated with two different scenarios and their results are compared.

System Description
Chaotic systems

Chaotic systems are deterministic systems with high complexity and irregular behavior and categorized as nonlinear dynamic systems. There are two common approaches to identify and measure chaos: graphical methods and quantitative methods (Wiggins, 1990; Sprott, 2003). Graphical methods such as time series and phase plots are visible but less accurate, while quantitative methods can determine the degree of chaos.

The Lyapunov exponent (LE) as the most important quantitative method that measures the sensitivity of initial conditions is a standard quantifier for determining and classifying the behavior of nonlinear systems. A wide range of LEs can be theoretically obtained, but the largest LE is of significance importance, which is calculated as follows (Sprott, 2003):

\[ \lambda_{\text{max}} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N-1} \ln \left| \frac{\Delta R_{n+1}}{\Delta R_n} \right| \]  

where \( \Delta R_n \) and \( \Delta R_{n+1} \) are the distances between two nearby trajectories at times \( n \) and \( n+1 \), respectively. If all LEs are negative, the system will be stable. In chaotic systems, at least one LE or the largest LE is positive.

Extended time-delay feedback control

During recent years, the method of time-delayed feedback to control a chaotic system has attracted a plenty of research interest (Fradkov and Evans, 2005; Pyragas, 1992). Assume a continuous-time system is described by Eq.(2) as follows:

\[ \dot{x}(t) = F(x, u), \]  

where \( x \) is an \( n \)-dimensional vector of state variables and \( u \) an \( m \)-dimensional vector of inputs (control variables). Pyragas (Pyragas, 1992) considered stabilization of a \( \tau \)-periodic orbit of the nonlinear system (2) using a simple control law described by Eq.(3) as follows:

\[ u(t) = K[x(t) - x(t - \tau)], \]  

where \( K \) is the feedback gain and \( \tau \) is a time-delay.

An extended version of the time-delayed feedback method is presented by Eq.(4) as follows (Fradkov and Evans, 2005):

\[ u(t) = K \sum_{k=0}^{M} r_k \left[ y(t - k\tau) - y(t - (k+1)\tau) \right], \]  

where \( y(t) = h(x(t)) \in \mathbb{R}^l \) is the observed output and \( r_k, k = 1, ..., M \) are tuning parameters. For \( r_k = r^k |x| < 1 \), and \( M \to \infty \) the control low (4) is represented by Eq.(5) as follows:

\[ u(t) = K[y(t) - y(t - \tau)] + Kru(t - \tau). \]  

Several studies have investigated the performance and limitations of the Pyragas methods (3, 5). Using a Pyragas controller, Ushio (1996) established proposed a simple necessary condition for stabilizability for a class of discrete-time systems (3). Nakajima (1997) proposed a proof for more general and continuous-time cases.
corresponding results for an extended control law (4) were presented in Konishi, Ishii and Kokame (1999) and Nakajima and Ueda (1998). Recently, Pyragas (Pyragas, 2001) suggested using the controller (5) with \( |K| > 1 \). In this method, the controller itself becomes unstable while stability of the overall closed-loop system can still be preserved.

**Model**

In this paper, a supply chain with two ordering policies is investigated which, like the beer distribution model, has four successive levels: factory, distributor, wholesaler, and retailer (Fig. 1). In this system, orders propagate from customers to factory and products flow from factory to customers.

Each level in the supply chain receives incoming products after a time delay from the time of placing an order. Meanwhile, a new demand is received. Based on their supply capacity, entities fulfill all or part of the backlog and current demand. Operations of each level are represented by Eqs. (6 & 7) as follows:

\[
\begin{align*}
    x_i(t+1) &= x_i(t) + o_i(t) - d_i(t), \\
    y_i(t+1) &= y_i(t) + o_i(t) - o_i(t-\tau),
\end{align*}
\]

where \( x_i(t) \) is the effective inventory (inventory level after fulfilling the backlog), \( y_i(t) \) the actual supply line (orders placed but not yet received), \( o_i(t) \) the demand, and \( \tau \) is a time delay between order placement and delivery.

The most important decision variable in the supply chain is the order quantity that has an essential role in its behavior. This paper examines two ordering policies. A well-known one is the smooth ordering policy whose decision equation is defined by Eq. (8) as follows:

\[
\begin{align*}
    o_i(t) &= \max \left[ 0, \hat{d}_i(t) + \alpha_i e_i^x(t) + \beta_i e_i^y(t) \right], \\
    0 &\leq \alpha_i \leq 1, 0 \leq \beta_i \leq 1,
\end{align*}
\]

where \( \alpha_i \) is the inventory adjustment parameter and \( e_i(t) \) the error between the actual inventory \( x_i(t) \) and the desired inventory \( \bar{x}_i(t) \):

\[
    e_i^x(t) = \bar{x}_i(t) - x_i(t).
\]

is the supply line adjustment parameter and \( e_i(t) \) the error between the actual supply line \( y_i(t) \) and the desired supply line \( \bar{y}_i(t) \):

\[
    e_i^y(t) = \bar{y}_i(t) - y_i(t).
\]

\[
\]where \( \bar{d}_i(t) \) is the demand forecast that is usually obtained from exponential smoothing (ES) forecast method:

\[
\hat{d}_i(t) = \theta_i d_i(t) + (1-\theta_i) \hat{d}_i(t-1),
\]

\[
0 \leq \theta_i \leq 1.
\]

\( \theta_i \) is a parameter which determines how fast expectation are updated.

A new ordering policy based on the ETDF control is used in the model:

\[
\begin{align*}
    o_i(t) &= \max \left[ 0, \hat{d}_i(t) + \alpha_i e_i^x(t) + \alpha_i^f(t) \right], \\
    \alpha_i^f(t) &= \alpha_i \left[ e_i^x(t) - e_i^x(t-1) + r_i^f \sigma_i^x(t-1) \right], \\
    \alpha_i^y(t) &= \beta_i \left[ e_i^y(t) - e_i^y(t-1) + r_i^y \sigma_i^y(t-1) \right].
\end{align*}
\]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( \lambda \leq 0 )</th>
<th>0\leq \lambda \leq 0.01</th>
<th>0.01\leq \lambda \leq 0.02</th>
<th>0.02\leq \lambda \leq 0.05</th>
</tr>
</thead>
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<tr>
<td>2325</td>
<td>477</td>
<td>543</td>
<td>6655</td>
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</tr>
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<td>734</td>
<td>485</td>
<td>1596</td>
<td>7185</td>
<td>2</td>
</tr>
</tbody>
</table>
\( r_i^a \) and \( r_i^\beta \) are tuning parameters which are adjustable. In this policy, the difference between two successive errors and its past orders are used to accelerate decision-making. \( \alpha \) and \( \beta \) are like the smooth ordering policy.

**Simulation**

Consider a supply chain with four levels. There are two scenarios for decision-making: smooth ordering policy and ES forecast method (Scenario 1), and ordering policy based ETDF control and ES forecast method (Scenario 2). It is assumed that all levels simultaneously use one scenario and their parameters are the same. Initial values and parameters are set according to Table 1. The model is simulated with the MATLAB software and in each scenario, 2000 data points are used to calculate the maximum LE.

Now with two scenarios, effects of inventory adjustment parameter and supply line adjustment parameter on the behavior of the supply chain are investigated. Assume that an ES forecast method is used in all levels and the supply line adjustment parameter is constant at 1.0. Change the inventory adjustment parameter from 0 to 1 and use two ordering policies. The chaotic behavior of the supply chain is studied through calculating the maximum LE. The results show that the ordering policy based on ETDF control (Scenario 2) is suitable, thus the behavior of the supply chain is stable in a greater range of \( \alpha \) (Fig. 2). Now, the inventory adjustment parameter is kept constant.

Fig. 1. The beer distribution model.

Fig. 2. The effect of inventory adjustment parameter.

Fig. 3. The effect of supply chain adjustment parameter.

Fig. 4. Comparison of all scenarios with \( \lambda_{\text{max}} < 0 \).

Fig. 5. Comparison of all scenarios with \( \lambda_{\text{max}} < 0.01 \).
(α - 0.7) and β is changed from 0 to 1 to study the effect of the supply line adjustment parameter on the supply chain (Fig. 3). The maximum LE of both ordering policies is positive in a large range, but with Scenario 2, the chaotic behavior of supply chain is less intense (Fig. 3). In addition, the stability range is larger with this scenario.

For greater certainty, maximum LE is recalculated by changing α and β with an increment of 0.1 from 0 to 1 in two scenarios. The number of Maximum LEs in different ranges is shown in Table 2. In the stable state (λmax < 0), scenarios are compared in Fig. 4. The chaotic behaviors of the supply chain with less intense (0 ≤ λmax < 0.01) are compared in Fig. 5. Simulation results indicate that once again Scenario 2 is the most suitable one.

Finally, the results of simulations show that the Scenario 2 is the most suitable scenario. In other words, the ordering policy based on ETDF control and the ES forecast method is effective in reducing the chaotic behavior of the supply chain.

CONCLUSION

A supply chain behaves as a nonlinear dynamics and may exhibit chaotic behavior. The ordering policy has the most important role in the behavior of the supply chain. The ordering policy based on ETDF control plays a crucial role in stabilizing its behavior. This policy speeds up the decision-making process.

The inventory adjustment parameter is an important decision variable and has a major role in controlling the inventory. Ordering policy based on ETDF control makes a better behavior of the supply chain in face of changes in the inventory adjustment parameter.

The supply line adjustment parameter is another decision variable which adjusts the discrepancy between actual and desired supply line. It is important in decision-making. Again, the ordering policy based on ETDF control (Scenario 2) is more appropriate.

Controlling chaotic behavior in the supply chain by other control methods such as robust control, adaptive control, and sliding mode control would be an interested area for future investigation.

REFERENCES

16. Thomsen, J.S., Mosekilde, E., & Sterman, J.D. Hyper chaotic phenomena in dynamic decision making. System Analysis and Modeling
