Linear Electron Accelerators Aimed at Environmental Protection

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The issue of environmental protection and sustainable use of natural resources becomes more and more acute and urgent. Environmental protection, sustainable use and reproduction of versatile natural resources, environmental enhancement are the main constituents of mankind survival in the World. The scope of economic operations and specific character of up-to-date technologies, especially in such industries as metallurgy and chemistry, necessitate dedicated measures aimed at environmental protection. As a result of the performed studies it has been demonstrated that it is possible to develop a linear accelerator of continuous action for use in industry and environmental protection.

Key words: Linear electron accelerator, Biperiodic slow wave system, Accelerating unit, Couple unit, shunt resistance.

Extreme intensification of existing industries and development of new engineering sectors require for improvement of known and creation of innovative procedures of waste processing. One of the promising trends of industrial waste processing is the application of radiation technology.

Radiation oxidation leads to intensive decomposition of pollutants, especially of organic wastes. In addition the processes of destruction, radiolysis and others are observed, which improve the treatment of effluent waters and waste gases. During radiation treatment good results are obtained with regard to disinfection of municipal, industrial and agricultural effluents (Toporkova et al., 2012).

Application of radiation sources for maintaining of environmental equilibrium puts forward peculiar requirements to their design (McKeown et al., 1978). Profitability, that is, low expenses, first of all cost of the facility itself and auxiliaries, and low operation costs, which predetermines quick return on investment. Reliability and high operation lifetime. Simplicity and ease of maintenance and control. Small sizes, which facilitates installation of radiation facilities in local biological protection.

While comparing radioisotope sources with electron accelerators most experts prefer accelerators, since capital investments for treatment facility with electron accelerator are by 7 times lower than those for facility with cobalt-60 and operation costs are by 2 times lower, respectively (Kon’kov, 1976).

For facilities with 0.4-1.0 MeV accelerator the economic effect is even higher. Reliability and simplicity of control for the facilities of both types are comparable. However, safety operation of the facilities with accelerators is significantly higher than that with radioisotope sources, especially with high power ones.

The disadvantages include small path of electrons with the energy up to 1 MeV in solids.
This disadvantage can be overridden by means of radiation of sprayed liquid, thin layer of free falling effluent water or preliminary foamed liquid. In addition, the use of slow wave radiation of accelerated electrons eliminates all difficulties related with limitation of radiated layer thickness.

While analyzing possibilities of pulse accelerators of direct action for waste treatment the most suitable is a transformer accelerator of horizontal layout for 2-3 MeV with beam power of about 100 kW. Estimated dimensions of such accelerator are as follows: diameter of about 2 m, length of 4-5 m. The required surface area including radiation protection is about 150 sq. m.

Application of resonant electron accelerators for effluent water treatment becomes possible after conversion from conventional pulse operation mode to continuous operation mode. Let us consider design peculiarities of linear electron accelerators (LEA), intended for environmental protection. Previous all resonant LEAs were based on round diaphragm-type waveguide (RDW), but recently, in addition to RDW, biperiodic slow wave systems (BSW), initially intended for linear proton accelerators, become widely spread.

A characteristic feature of BSW is high effective shunt resistance, by 1.5-2 times higher than that of RDW. This permits to decrease longitudinal dimensions of LEA with steady or slightly increased transversal dimensions of accelerator slow wave system. This circumstance caused occurrence of a class of linear accelerators based on BSW, operating in standing wave mode.

At present a wide set of LEA was develop for various industries: for medicine (Reitsamer et al., 2008; Zhang et al., 2013), non-destructive testing (Vikulov et al., 1979), chemical industry (Pikaev, 1995) and others (Machi, 2009; Zabaev, 2008). Each accelerator, depending on the scope of its application, has its own peculiarities, but all these devices operate in pulse mode and has low beam power, thus none of them can be applied profitably for environmental protection.

In order to eliminate this drawback it is required to develop LEA based on BSW pf continuous action. Such accelerator with output energy of 2-3 MeV and beam power of 100 kW will not exceed the following dimensions: length of 1 m, height of 1 m, width of 0.5 m, which is significantly lower than those of transformer accelerator. In addition, the LEA design of continuous action is slightly simplified in comparison with pulse resonant accelerator, since no modulator of super high frequency (SHF) source is required.

**EXPERIMENTAL**

**General provisions**

Conversion to continuous operation mode creates some difficulties, the first of which is low value of accelerating field amplitude, requiring for application of external focusing tools. The second difficulty is related with limitation of longitudinal dimensions of accelerating structure, since the requirement of small size is typical for all industrial accelerators, and this causes certain difficulties in computation of particle dynamics. The third and the main difficulty is stipulated with high heating of slow wave system due to significant dissipation of SHF energy within the walls of accelerating units. Temperature increase of the elements of slow wave system leads to deformation of the units and variation of resonant frequency and accelerating field amplitude in BSW.

In order to suppress this phenomenon specialized systems of frequency adjustment and thermal mode control are applied (McKeown et al., 1976; McKeown et al., 1978), which provide accelerator working both during start-up and operation (McMichael et al., 1979). Operation costs of control systems can be significantly reduced with available information concerning behavior of BSW during heating and provisions for readjustment of SHF generator frequency. With this aim it is required to determine temperature deformations of slow wave system, occurring during LEA operation.

While developing LEA of continuous action, certain problems are solved, typical for any linear accelerator, either proton or electron. At first geometrical dimensions of slow wave system are determined, then, on its basis, designing and arrangement of LEA are executed. In order to determine geometrical dimensions of BSW it is required to be aware of main electrodynamic characteristics of slow wave system as a function of its geometrical dimensions.
It would be reasonable to represent the main electrodynamic characteristics in parameterized form, which would facilitate application of the obtained data in any frequency range. Simulation of BSW by means of a chain of coupled resonators makes it possible to reveal functional dependence of the dispersion characteristics of slow wave system on its geometrical dimensions.

Initial simulation of BSW by means of a chain of coupled resonators was performed in (Nishikava et al., 1966), where the authors in their conclusions relied on the equations for a chain of resonators (Slater, 1948) and procedure of solution of these equations described elsewhere (Allen and Kino, 1960). In the latter work the authors obtained dispersion relations for a chain of cylindrical resonators coupled via narrow gaps, however, the influence of accelerated beam on these relations was not considered.

It should be mentioned that previous and subsequently developed works (Nagle et al., 1967; Knapp et al., 1968), where biperiodic structures are simulated by a chain of coupled radiotechnical circuits, made it possible to determine a set of characteristics of these systems and to compute the particle dynamics in them. Nevertheless, such representation of biperiodic systems is characterized with a set of principal limitations. In particular, using a contour model it is necessary to preset parameters of single units, such as effective shunt resistance, high quality, eigen frequency and distribution of electromagnetic field in them, which should account for existence of gaps of the couple.

Determination of these parameters is an autonomous problem but does not imply unsuitability of this model, on the contrary, at the stage of adjustment of pilot structures this model is the most acceptable, since all characteristics for it are taken directly from experiment. However, at the stage of designing and selection of the structure geometry the model of coupled contours is inferior to the model of coupled resonators, since the latter model makes it possible to couple main electrodynamic characteristics of slow wave system with its geometrical dimensions.

Equations for a chain of coupled resonators

Let us obtain equations interrelating geometrical dimensions of BSW and values of accelerated currents with its electrodynamic characteristics. With this aim we will consider an arbitrary resonator with certain volume \( V \) limited with closed surface \( S \). This surface is metallic shell of the resonator with holes for coupling with other resonators or inlet waveguide duct. We consider the problem of detection of electromagnetic fields established in resonator at frequency \( \omega_2 \).

Since the internal resonator surface can be sufficiently complex, then the accurate solution of the Maxwell equations for the considered volume is not possible. Hence, let us present the required solution as the sum of normal oscillations characterized with orthogonal property.

It is known (Slater, 1948) that any solution of the Maxwell equations for resonator in general case contains solenoidal (vortex) and potential (gradient) constituents. Let us be confined only with the solenoidal constituent, which is usually considered as “radiation field”.

Let us select certain set of solenoidal functions \( (E_m, H_m) \), (where \( m = 1, 2, \ldots \)), according to which the required fields \( E \) and \( H \) are decomposed in the resonator. The selected equation set satisfies wave equation and homogenous boundary conditions. Therefore, the set of solenoidal functions is the solution of the Maxwell equations in the resonator, the surface of which is an ideal conductor. Thus, it is possible to state that in these assumptions the selected set forms complete orthonormal set of functions (Mashkovtsev et al., 1966)

\[
\int_V E_k E_m \, dV = \int_V H_k H_m \, dV = \delta_{km} = \begin{cases} 0, & k \neq m \\ 1, & k = m \end{cases}
\]

In this case it is possible to obtain the following set of equations for the required electric field of the considered resonator:

\[
\begin{align*}
\frac{d^2}{dt^2} \int_V E_k E_m \, dV &+ \omega_0^2 \int_V E_k E_m \, dV = -\frac{\alpha_m}{\sqrt{\varepsilon \mu}} \int_S [nE] H_m \, dS \\
- \frac{1}{\varepsilon} \frac{d}{dt} \int_V J_k E_m \, dV &- \frac{\alpha_m}{\sqrt{\varepsilon \mu}} \int_{S_{hc}} [nE] H_m \, dS.
\end{align*}
\]

Here \( E_m, H_m, \omega_m \) are the eigen functions and eigen frequencies of the resonator; \( J \) is the vector of current density in an arbitrary point of the resonator space; \( t \) is the time; \( S_{hc} \) is the
cumulative surface area of couple holes of the resonator; \( \varepsilon, \mu \) are the absolute dielectric and magnetic permeability, respectively.

Similar equation can be obtained also for magnetic field. The obtained system (2) can be generalized also for the case of a chain of coupled resonators, forming BSW, herewith the appropriate variables get the index \( n \) (resonator number):

\[
\frac{d^2}{dt^2} \int \mathbf{E}_n \cdot d\mathbf{v} + \omega_0^2 \int \mathbf{E}_n \cdot d\mathbf{v} = \frac{\omega_0^2}{\sqrt{\mu \varepsilon}} \int [n\mathbf{E}_s] \cdot \mathbf{H}_{nm} d\mathbf{S} - \frac{1}{\varepsilon} \int \mathbf{J}_s \cdot d\mathbf{v} - \frac{\omega_0^2}{\sqrt{\mu \varepsilon}} \int [n\mathbf{E}_s] \cdot \mathbf{H}_{nm} d\mathbf{S} \]

The order of the system is determined as the product of the number of resonator chain by the number of selected functions of the system \((\mathbf{E}_n, \mathbf{H}_{nm})\) in the expansion of fields \(\mathbf{E}\) and \(\mathbf{H}\).

The set of equations (3) is the required system for the chain of coupled generators. Solution of this system makes it possible to determine in general case the electrodynamic characteristics of slow wave structure, formed by the chain of coupled resonators. They are as follows: dispersion, field distribution by structure, characteristics of slow wave structure, formed by couple holes of the resonators between each other. Then the required fields in the \(n\)-resonator of slow wave structure can be presented as follows:

\[
E_n(r, t) = \text{Im}\{E_n(r)e^{-i\omega t}\} = \sum_{m=1}^{\infty} \text{Im}\{V_{nm} E_{nm}(r)e^{-i\omega t}\} \quad \ldots(4)
\]

\[
H_n(r, t) = \text{Im}\{H_n(r)e^{-i\omega t}\} = \sum_{m=1}^{\infty} \text{Im}\{I_{nm} H_{nm}(r)e^{-i\omega t}\} \quad \ldots(5)
\]

where \(V_{nm}, I_{nm}\) are the unknown amplitude coefficients of expansion of the fields \(\mathbf{E}\) and \(\mathbf{H}\); \(\mathbf{r}\) is the radius-vector of the considered point in the \(n\)-resonator.

Then, with consideration for Eqs. (1), (4), and (5) the left-hand side of Eq. (3) can be transformed as follows:

\[
\frac{d^2}{dt^2} \int \mathbf{E}_n \cdot d\mathbf{v} + \omega_0^2 \int \mathbf{E}_n \cdot d\mathbf{v} = \text{Im}\{(\omega_0^2 - \omega^2)(V_{nm} e^{-i\omega t})\}
\]

The first term in the right-hand side of Eq. (3) reflects losses in the resonator walls and can be expressed via the value of its own quality \(Q_{nm}^2\):

\[
Q_{nm}^2 \text{Im} \int \mathbf{E}_n(r) \mathbf{H}_{nm} d\mathbf{S} = \omega^2 (i - 1) \frac{1}{Q_{nm}^2} V_{nm} \quad \ldots(7)
\]

The integrals over the surface area of couple holes \(S_{hc}\) in Eq. (3) reflects excitation of the \(n\)-resonator by electromagnetic fields in couple holes with adjacent or with inlet waveguide duct, if it is supplied to the \(n\)-unit. In the considered case the electric couple between the resonators can be neglected, and the integral form of the couple hole surface is as follows:

\[
\int_{S_{hc}} [n\mathbf{E}_n] \cdot \mathbf{H}_{nm} d\mathbf{S} = \int_{S_{hc}} [E_{gn} H_{nm}] n d\mathbf{S} \quad \ldots(8)
\]

Substituting the obtained Eqs. (6), (7), and (8) into Eq. (3) we obtain the set of equations with regard to the unknown coefficients \(V_{nm}\)

\[
(\omega_0^2 - \omega^2)V_{nm} = \omega^2 (i - 1) \frac{1}{Q_{nm}^2} V_{nm} - \frac{\omega_0^2}{\sqrt{\mu \varepsilon}} \int_{S_{hc}} [E_{gn} H_{nm}] n d\mathbf{S} \quad \ldots(9)
\]

The variables to be specified and determined in Eq. (9) include tangential constituent of electric field on couple hole \(E_{gn}\). The analyzed BSW contain axial symmetrical resonators, in side walls of each of them narrow gaps are cut in the direction of azimuth coordinate.

Since the gap width is significantly lower than the wave length, it makes possible to represent the gap as transmitting line without losses, short-circuitated at the ends. Along such line a wave of \(T\) type is transmitted, herewith in the left and right planes of the gap only tangential component of electric field exists. Such assumption is in direct logical correspondence with neglect of the field potential part.

The gap distributed capacity is determined from the solution of electrostatic problem in the plane perpendicular to the propagation direction of wave.
Solution of equation set for a chain of coupled resonators

Solving Eq. (9) it is assumed that eigen parameters of the resonators, such as eigen frequencies, eigen qualities, eigen electric and magnetic fields are known.

Aiming at their solution let us consider an axial symmetric resonator with ideal conducting surface. We will be confined with the lowest type of oscillations. Due to axial symmetry of magnetic field and , in order to calculate eigen value and eigen function of main type of oscillations it is possible to apply the finite difference method (Wasow and Forsythe, 1963) for direct numerical integration of the Maxwell equations. It is known that the fields in resonant chamber in vacuum are determined by the wave equation. Since and , and at the substitution we finally obtain:

\[
\frac{\partial^2 F}{\partial r^2} \frac{1}{r} \frac{\partial F}{\partial r} + \frac{\partial^2 F}{\partial z^2} + k^2 F = 0
\]  

...(10)

In order to solve Eq. (10) it is necessary to determine boundary conditions. For ideal conducting metal surfaces of the resonator these conditions are written as follows: \( \frac{\partial F}{\partial n} = 0 \), where \( n \) is the unit normal vector to the resonator surface.

The following relations will be valid in the hole of drift tube between the resonators: \( F = 0 \) in the case of excitation of \( \pi \) – type of oscillations; \( \frac{\partial F}{\partial n} = 0 \) in the case of excitations of \( \phi \) – type of oscillations; on the resonator axis.

In calculations the resonators are considered symmetrical with regard to the plane passing via the resonator center perpendicular to its axis. As a consequence, the region considered upon calculation of the field can be reduced to one half, and cylindrical symmetry of the resonator reduces this region to one quarter.

For numerical solution of the problem Eq. (10) is presented in difference form. First of all, rectangular grid with increment \( h \) is imposed onto transversal cross section of the resonator. Then, the boundary surfaces of the resonator are approximated by polygonal line, which coincides with actual boundary and passes outside. Then, the difference equations, expressing the interrelation between the values on the polygonal line and in adjacent points, are arranged to satisfy boundary conditions on actual boundary.

After that the differential Eq. (10) is substituted with the set of difference equations for all grid nodes. In order to solve the obtained set of algebraic equations it is convenient to apply the over-relaxation method. All grid is passed subsequently, point by point, with substitution of \( F^{(n)} \) in each node with newly obtained \( F^{(n+1)} \), according to the following equation (Wasow and Forsythe, 1963):

\[
F^{(n+1)} = F^{(n)} + \Omega k^2 \left( \frac{1}{\rho} \frac{\partial F^{(n)}}{\partial \rho} + \frac{\partial^2 F^{(n)}}{\partial z^2} + k^2 F^{(n)} \right)
\]  

...(11)

where \( \Omega \) is the convergence coefficient, its optimum value is in the range of 0.15-0.5.

The eigen value \( k^2 \) is determined by the term (Hoyt et al., 1966) of variation problem

\[
k^2 = \frac{\int_S \frac{1}{\rho} \left( \frac{\partial F}{\partial \rho} \right)^2 + \left( \frac{\partial F}{\partial z} \right)^2 d\rho dz}{\int_S \frac{F^2}{\rho} d\rho dz}
\]  

...(12)

Since the solution of this problem is reduced to determination of not only eigen functions but also of eigen value, then double iteration procedure is applied, its essence is as follows: setting certain initial approximation of eigen value according to Eq. (11), using the over-relaxation method, the eigen value distribution in the resonator chamber is determined. Then, using Eq. (12), new eigen value is calculated and the procedure is repeated until the preset accuracy is achieved with regard to simultaneous determination of \( F \) and \( k^2 \). The work (Young et al., 1963) proved that this procedure converges and provides the lowest possible eigen value.

After determination of the wave number and magnetic field distribution over the unit volume it is possible to calculate electrodynamic characteristics of accelerating resonators by direct integration. In particular, the power of losses is determined as follows:

\[
P = \frac{1}{2} R_s \int_S H_s^2 \, ds \approx 2 \pi R_s h \sum_{i=1}^n \frac{F_i^2}{\rho_i}
\]

where \( R_s \) is the surface resistance of copper; \( \rho_i \) is the radius between the considered point and axis. Summation is performed over all boundary points.
An important parameter of accelerating structures, operating in the standing wave mode and characterizing their sensitivity to perturbations, is the dispersion characteristic. The dispersion characteristic, corresponding to fundamental harmonic for BSW, is determined as phase shift of oscillations of electromagnetic field by the element of structure periodicity:

\[ \phi = \frac{2\pi \lambda}{\lambda \beta} \]

where \( L \) is the length of accelerating structure, \( \lambda \) is the wave length, \( \beta \) is the variation rate of the phase of high frequency field in the units of speed of light.

In order to determine the dispersion characteristic it is required to apply the values of quality and magnetic field at the level of couple gap and set the determiner of the obtained system equal to zero (McKeown et al., 1976). Real parts of the roots will correspond to the frequencies of 0 and \( \pi \) oscillations, on the basis of which it is possible to determine the coefficient of couple as follows:

\[ k_c = \frac{\omega^2 - \omega_0^2}{\omega^2 + \omega_0^2} \]

RESULTS

As a consequence of the performed calculations the families of parametric plots and tables were obtained for main electrodynamic properties of the accelerating units of BSW, on the basis of which it is possible to estimate the efficiency of this or that design of slow wave system.

The equations have been obtained, where all main electrodynamic properties and all geometrical dimensions are normalized with respect to wave length. Thus, it is possible to apply these data for plotting of curves and tables, which would be valid for any frequency range.

In order to determine the dispersion characteristic it is necessary to solve the set of linear algebraic equations. These equations include distributed capacity of couple gap, which was determined from the solution of electrostatic problem as the capacity of two infinitely thin plates. The calculations demonstrated that the distributed capacity of the couple gap equals to 5.6 F/m, if the couple gap is remote from the unit side wall, and 11.2 F/m, if the gap is close to the side wall. These values slightly differ from the calculated ones, since they do not consider for the influence of all geometric dimensions of the couple gap on the distributed capacity.

The equations are obtained facilitating calculations of main electrodynamic characteristics of BSW.

The influence of current loading in BSW is taken into account with the lowest harmonic of current beam.

The equations are obtained facilitating calculations electric field distribution along the length of slow wave system depending on inlet power and permitting consideration for the influence of the geometry of couple gap on the dispersion characteristic of BSW.

Possibility of parametrization of electrodynamic characteristics of BSW has been proven.

Series of parametric plots and tables have been obtained aiming at engineering calculations of BSW and its adjustment to operation frequency.

DISCUSSION

BSW accelerators, operating in the standing wave mode, present a chain of coupled resonators, which can be excited at strictly defined frequencies. Analytical interrelation between the excitation frequency and phase shift by periodicity element is known as the dispersion dependence, and its graphical interpretation as the dispersion curve. The dispersion curve of accelerating section operating at standing wave is a series of points corresponding to resonant frequencies of slow wave system equaling to the number of resonators in the slow wave structure.

It has been demonstrated theoretically and experimentally (Knapp et al., 1968) that the BSW sensitivity to the current loading and other types of detuning decreases, and this decrease is the lower, the more distant is the operating type of oscillations from adjacent oscillations. Therefore, in order to determine the BSW sensitivity to excitation factors it is necessary to know the dispersion dependence.

It is known (Wasow and Forsythe, 1963) that the dispersion curve in general form is split...
into two branches separated by the stop band. Existence of the stop band leads to inequality of electric field distribution in accelerating units and decrease in effective shunt resistance due to occurrence of electric field in couple units.

The BSW adjustment consists of elimination of the stop band by variation of certain dimensions of units and fulfilling of the term , where is the frequency of supplying generator, is the frequency of –type of oscillations. Since the adjustment of the slow wave structure is related with labor and time consuming activities, this procedure was supported by calculated and arranged tables and plots, which facilitate determination of eigen frequencies of accelerating units and couple units.

**CONCLUSIONS**

Let us make some final conclusions concerning selection of parameters of accelerating units and couple units of BSW:

- Frequency of supplying generator is usually preset;
- Shape of peripheral boundary of accelerating units is stipulated by approved production technology;
- Shapes and dimensions of couple units are determined depending on requirements for effective shunt resistance, longitudinal and transversal dimensions of BSW, conditions of heat transfer and design of cooling system and output parameters of selected source of SHF power.

Period of BSW structure is determined on the basis of the following conditions. If the couple units by their design are located outside of the axis of slow wave system, then for the length of accelerating unit the following expression is valid: . If the couple units are located on the axis of slow wave system, then the BSW period is .

One of the most important dimensions of accelerating units, which determines overall efficiency of accelerating structure, 9is the following parameter: , where is the radius of floating channel hole. This value is selected from the condition of obtaining of maximum shunt resistance at preset phase speed of accelerating wave. Of the same importance is the radius of floating channel hole, which is determined by longitudinal and radial dynamics of electrons. The variable should be as low as possible, since the effective shunt resistance increases sharply with decrease in .

The drift tube thickness is selected with account for several factors. Thus, with increase in the drift tube thickness the electric strength of resonator increases and the effective shunt resistance decreases. Therefore, in designing of BSW it is always necessary to account for a chance of breakdown in accelerating unit. The wall thickness between the resonators is selected with consideration for heat transfer, preset coefficient of couple and adjustment procedure.

The performed studies demonstrated a possibility of development of LEA of continuous action with the following characteristics: the energy of accelerated electrons of 0.5-0.8 MeV; the current of accelerated electrons of 20 mA; the generator operating frequency of 2450 MHz; the generator SHF power 40 kW, the injection voltage of 20 kV; the slow wave structure length of m.

We plan to perform further study of optimization of thermal mode in continuous accelerators and application of possibility of grouping and focusing of accelerated electrons by high-frequency fields of BSW with standing wave (Novozhilov et al., 2015).

**REFERENCES**


17. Young, D E; Christian, R.S; Curtis, C D; Edwards, T W; Kriegler, F J; Mills, F E; Morton, P L; Swenson, D A; Van Bladel, J. Design studies of proton linear accelerators From the International Conference on High Energy Accelerators, Dubna, U.S.S.R., 1963; p.p.454-461. DOI: 10.2172/4632371


