

## Modeling of Contact Connecting Layer of Compound Rod in Designing of Adhesive and Adhesive Mechanical Joints

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**A dynamic analytical model of the compound, which allows taking into account the different types of the non-linearity contact connecting layer. Taking into account the accepted assumptions there longitudinal and bending vibrations of distinctive feature three-layer rod are described.**

**Key words:** Dynamic model of bonded-mechanical compound, Fluctuations differential element, contact connecting layer.

Bearing layers (both permanent and variable cross-section), are usually taken as linearly elastic. Connecting layer, which provides the deformation compatibility of the bearing layers, is considered as without inertia. The thickness of the contact layer can also be accepted as variable. In addition, it is assumed that the movements in the bearing layers are described by equations of S.P.Timoshenko<sup>1</sup>, which takes into account the inertia of the cross-section rotation, and when the rod is bent, its section is not perpendicular to the center line, turning for the value of some shear angle. The latter assumption leads to wave equations of vibrations of rods.

To describe the longitudinal and flexural vibrations of the differential element three-layered rod can use the following equation (see Fig.1).

$$\left\{ \begin{array}{l} (F_1)\bar{d}x - 2b_y\tau dx = m_1 a_{x1}; \\ (F_2)\bar{d}x + 2b_y\tau dx = m_2 a_{x2}; \\ -(Q_1)\bar{d}x - 2b_y\sigma dx = m_1 a_{z1}; \\ -(Q_2)\bar{d}x + 2b_y\sigma dx = m_2 a_{z2}; \\ (M_1)\bar{d}x - Q_1 dx - b_y H_1(x)\tau dx = J_1 a_{(\delta)}; \\ (M_2)\bar{d}x - Q_2 dx - b_y H_2(x)\tau dx = J_2 a_{(\zeta)}; \end{array} \right. \dots(1)$$

Here by the symbol (...)’ partial derivatives are determined  $E, F_1, F_2$  - longitudinal forces for elements 1 and 2 rod,  $Q_1, Q_2$  - shear forces for elements 1 and 2 rod,  $M_1, M_2$  - bending torques for elements 1 and 2 rod,  $\tau, \sigma$  - tangential and pulling stress in the adhesive layer,  $m_1, m_2$  - the masses of elements 1 and 2 rods,  $J_1, J_2$  - mass torques of element inertia 1 and 2 rods relative to the axis C,  $0_{E1}, 0_{E2}$  -longitudinal accelerations of 1 and 2 rods,  $a_{z1}, a_{z2}$  - transversal accelerations of 1 and 2 rods,  $0_{(d)}, a_{(z)}$  - angle accelerations of elements 1 and 2 rods round the axis y,  $dx$  - the length of the compound element of the rod,  $b_y$  - the half of the width of the compound rod element,  $H_1(X), H_2(X)$  - variable widths of elements 1 and 2 rods.

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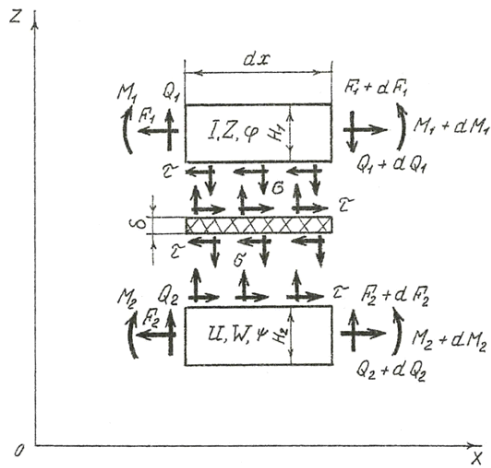


Fig. 1. Differential element of three-layered rod

Let  $I, U$  – longitudinal displacements of 1 and 2 rod,  $Z, W$  – their vertical displacement,  $\varphi, \psi$  – rotation angles of cross section. Then it is true that –

$$\begin{cases} F_1 = 2b_y H_1 E_1(I); F_2 = 2b_y H_2 E_2(U); \\ Q_1 = -(2G_1 b_y H_1 / K_1)[(Z)^\square - \varphi]; Q_2 = -(2G_2 b_y H_2 / K_2)[(W)^\square - \psi]; \\ M_1 = J_1(\varphi)^\square; M_2 = J_2(\psi)^\square \end{cases}$$

here  $E_1, E_2, G_1, G_2$  – elasticity modulus during stretching and shearing for 1 and 2 rod respectively,  $K_1, K_2$  – coefficients, which depend on their cross section form.

In modeling the contact of the compound layer, as a rule, a number of simplifications are introduced:

- constituents of shear stresses in other directions ( $\tau_{xy}$  and  $\tau_{yz}$ ) are small, i.e. can be neglected;
- a connecting layer (due to smallness of its thickness  $d$  in comparison with the width of the connection) is in deformed flat condition;
- longitudinal deformation (along the  $x$  axis) is absent;
- stresses across the thickness of the contact layer is not changed [2].

The calculations can consider three models of contact connecting layer:

- linear-elastic layer, which simulates the behavior of rigid connecting elements;
- elastic-plastic layer with hardening, modeling the character of loading of rigid polymeric binders;

- visco-elastic connecting layer, which simulates the behavior of the majority of modern structural adhesives.

For the latest model of the relationship between stress  $\sigma, \tau$  and deformations  $\varepsilon, \gamma$  can be found using the Maxwell model –

$$\begin{cases} (\varepsilon)_t^\square = \left[ \frac{1-2\mu}{2G(1-\mu)} \right] (\sigma)_t^\square + [(1-2\mu)/(1-\mu)\eta] \times \\ \quad \times [\sigma + E_0 \sigma / 2G] - E_0 \varepsilon / \eta; \quad \dots(2) \\ (\gamma)_t^\square = (1/G)(\tau)_t^\square + 3\tau / \eta - E_0 \gamma / \eta + E_0 \tau / G\eta \end{cases}$$

Where by the symbol  $(\dots)_t^\square$ , the partial derivative are determined  $\mu, m$  – Poisson ratio of the connecting layer,  $E_{(\infty)}$  – displacement modulus and viscoelastic modulus of deformation constituent,  $\eta$  – relaxation viscosity coefficient of deformation constituent.

Deformation of the connecting layer is expressed through the displacement and rotation angles of the bearing layers' cross sections –

$$\varepsilon = (Z - W) / \delta; \quad \gamma = (I - U + H_1 \varphi + H_2 \psi) / \delta \dots(3)$$

where  $\delta$  – the thickness of the contact connecting layer.

The modulus of highly elastic constituent of deformation and coefficient of relaxation viscosity of the adhesive layer can be found by using a simplified method, based on the experiment by stretching of the overlapped joint at different speeds.

The initial conditions for the system of equations (3) have the form –

$$I(x) = U(x) = Z(x) = W(x) = \varphi(x) = \psi(x) = 0 \quad t=0 \quad \dots(4)$$

For stresses in the connecting layer at  $t = 0$  we have –

$$\sigma(s) = \tau(x) = 0 \quad \dots(5)$$

The boundary conditions can be written:

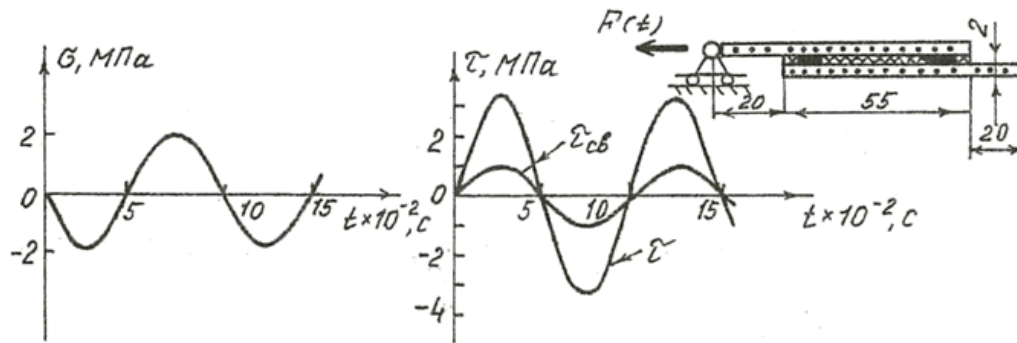
- for the free ending, the rods' forces and torques are equal to zero; for example, for the left ending of the rod 1 –

$$F_1|_{x=0} = Q_1|_{x=0} = M_1|_{x=0} = 0 \quad \dots(6)$$

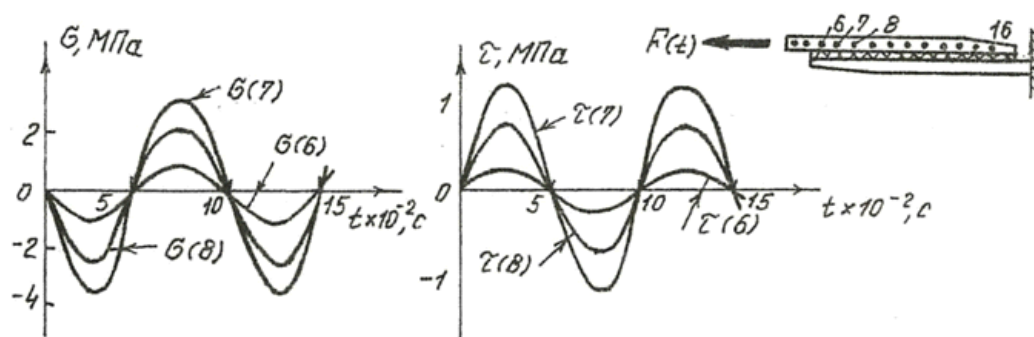
and at the swivel support

$$Z|_{x=0} = M_1|_{x=0} = 0 \quad \dots(7)$$

System of equations of dynamic equilibrium in terms of displacements, together with initial and boundary conditions can be integrated using the numerical method of finite differences. For this purpose, along the length of the joint 1 grid with  $NC$  step is introduced and differential element is replaced by a finite one. For reasons of the best approximation of differential equations of



**Fig. 2.** Time changes of normal and shear stresses in the left boundary section of the connecting layer of the combined joint; welding points with diameter  $d = 5$  mm located at a distance of 5 mm from the ends,  $E_1 = E_2 = 6860$  MPa;  $H = 0,5$  mm;  $E = 350$  MPa;  $E_{(\infty)} = 200$  MPa;  $G_{CB} = 264$  MPa;  $d = 0,5$  mm



**Fig. 3.** The dependence of the stress at the nodes of connecting layer on time during overlapping with beveled edges;  $H(6) = 0,25$  mm;  $H(7) = 0,5$  mm;  $H(8) = 0,75$  mm;  $E_1 = E_2 = 7000$  MPa;  $E = 350$  MPa;  $E_{(\infty)} = 200$  MPa;  $d = 0,5$  mm

different scheme, grid nodes should be selected in the centers of gravity of the elements.

As a result of solving of linear algebraic equations system desired values of displacements are defined, and for the calculation of both high and low-frequency oscillations implicit scheme is suitable.

The disadvantages of the discussed solution techniques are rather cumbersome calculation algorithm and the need to solve large systems of equations at each level time.

Examples of the evaluation results of combined joints' loading, obtained using this method are shown in Fig. 2 and Fig. 3.

The analysis of the dynamic processes' adequacy, obtained by calculation, the actual processes in the compounds was carried out by indirect evidence: the spectrum of natural frequencies, the level of vibration amplitudes, the

width of the resonance peaks, the damping of the oscillations.

In addition to the resonant modes of the samples the behavior under impact loading was studied. The oscillation process in this case was recorded by force transducers and recorded with an oscilloscope. Good agreement between calculated and experimental processes fluctuations was received.

Viscoelasticity characteristics were determined from tensile tests of the models of adhesive joints of with the length of 10 mm with different rates of loading<sup>3-10</sup>.

The method of longevity calculation was based on the linear hypothesis of damage summation in the most dangerous section of the connecting layer. The function of the stress amplitudes' distribution was built with the help of the method of complete cycles. Here we used the

transition from the calculation cycle of the stress changes to equally dangerous, for which the fatigue curve is known.

A comparison of the calculated data and the values of durability with calculation results and tests was conducted<sup>11-17</sup>. The discrepancy between the calculated and actual longevity was 30 ... 40%.

This technique is used to predict the longevity of model compounds with the structural and technological defects. For its implementation the base of fatigue curves of models compounds is needed.

### CONCLUSIONS

1. The model of the composite rod of varying degrees of complexity within the design calculations of adhesive and adhesive-mechanical joints, apparently, is a base because it gives an opportunity to analyze the workload of the connecting assembly according to the basic design parameters.
2. The results of composite rod model calculations should be systematized for making concrete recommendations for the design.
3. Calculated possibilities of the model of dynamic effects and durability evaluation are also used to clarify the relationship between the durability of standard compounds and structural and technological defects.

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