

Analysis of the Decoder's Operation at Correcting Single Errors Using the Triple Code

Igor Mikhailovich Beluchenko, Vyacheslav Nicolavich Zinoviev,
Yuri Veniaminovich Strenakuk and Vladimir Mikhailovich Vatutin

Financial and Technical Academy, Moscow Region, 141070, Korolev, Gagarin Street, 42, The Russian Research Institute of Space Device Engineering, 111250, Moscow, Aviamotornaya Street, 53.

doi: <http://dx.doi.org/10.13005/bbra/1463>

(Received: 27 September 2014; accepted: 10 October 2014)

The article deals with the singularities of the triple code application at transmission of digital information; shows the possibility of anti-jam protection of the triple code using methods developed for the binary code, in particular, methods of encryption based on the generator matrix and generator polynoms; and shows that methods of syndrome decoding are applicable for channel decoding.

Key words: Communication channels, Encryption, triple polynom.

The problem of reproduction and development of devices with the logic of interpretation of the concept of information for PC computations in the triple numerical system or by means of computing devices with triple memorizing elements occurred in the early 1970s. The prospects of devices using triple logics were doubted based on the assumption that information should be represented in the binary logic and the binary numerical system only. Developing the determined (non-probabilistic) combinatory approach of A.N. Kolmogorov to the quantitative representation of information¹ in this task, we come to another conclusion-the variants of integration of a message in the binary logic and the binary numerical system are reproduced by the results of similar integration

in the triple logic and the triple numerical system. At that, the more general triple coding allows representing the formation of information representations more accurately and fully, as unlike the binary coding, it excludes the rounding errors and provides for independent selection of the code value for the case of equivalent alternatives.

The methodology of formation of digit position anti-jam in the triple code

The issues of jam-proof coding are urgent for the contemporary media of data transmission in telecommunication systems⁴⁻¹³.

Let us find the syndromes of the errors. The type "1" errors: $S_1=101$; $S_2=011$; $S_3=110$; $S_4=111$; $S_5=100$; $S_6=010$; $S_7=001$.

The type "2" errors: $S_8=202$; $S_9=022$; $S_{10}=220$; $S_{11}=222$; $S_{12}=200$; $S_{13}=020$; $S_{14}=002$.

We will make a type "1" error in the third digit position and a type "2" error in the fifth position.

2	1	2*	0	2	0	0
0	0	1	0	0	0	0
2	1	0	0	2	0	0

* To whom all correspondence should be addressed.

$$\begin{matrix} 2 & 1 & 2 & 0 & 2^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 & 0 \end{matrix}$$

The obtained messages are multiplied by the matrix \mathbf{H}^T .

$$S = \left| \begin{array}{c} 101 \\ 011 \\ 110 \\ 111 \\ 100 \\ 010 \\ 001 \end{array} \right| \times \begin{array}{c} 2100200 \\ 2100200 \\ 2100200 \end{array} = (1,1,0)$$

is the syndrome of the type “1” error in the third digit position “ S_3 ”.

$$S = \left| \begin{array}{c} 101 \\ 011 \\ 110 \\ 111 \\ 100 \\ 010 \\ 001 \end{array} \right| \times \begin{array}{c} 2120100 \\ 2120100 \\ 2120100 \end{array} = (2,0,0)$$

is the syndrome of the type “2” error in the fifth digit position “ S_{12} ”.

Correction of the type “1” error in the third digit position of the received message and the type “2” error in the fifth position.

2	1	0	0	2	0	0
0	0	1	0	0	0	0
2	1	2	0	2	0	0

$$\begin{array}{ccccccc} \underline{2} & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ \hline 2 & 1 & 2 & 0 & 2 & 0 & 0 \end{array}$$

Let us consider the jam-proof coding using a generator polynomial.

We start with the example for the same code (4). The generator polynomial $g(x)=x^3+x+1$. The procedures of division are carried out using operations of field GF(3). The information message of type $m(x)=a_3x^3+a_2x^2+a_1x^1+a_0x^0$, in which the ratios a_i belong to the set (0,1,2), is multiplied by x^3 . Then, the polynomial $a_3x^6+a_2x^5+a_1x^4+a_0x^3$ is divided by the primitive (indivisible) polynomial $g(x)$.

Example. $m(x)=1x^3+2x^2+2x^1+1$.

$$\begin{array}{r} x^6 + 2 \cdot x^5 + 2 \cdot x^4 + x^3 + 0 \cdot x^2 + 0 \cdot x + 0 \\ x^6 + 0 \cdot x^5 + 1 \cdot x^4 + 1 \cdot x^3 \end{array} \left| \begin{array}{l} x^3 + 0 \cdot x^2 + x + 1 \\ x^3 + 2 \cdot x^2 + x + 1 \end{array} \right.$$

$$\hline 2 \cdot x^5 + 1 \cdot x^4 + 0 \cdot x^3 + 0 \cdot x^2$$

$$2 \cdot x^5 + 0 \cdot x^4 + 2 \cdot x^3 + 2 \cdot x^2$$

$$\hline x^4 + 1 \cdot x^3 + 1 \cdot x^2 + 0 \cdot x$$

$$x^4 + 0 \cdot x^3 + 1 \cdot x^2 + 1 \cdot x$$

$$\hline 1 \cdot x^3 + 0 \cdot x^2 + 2 \cdot x + 0$$

$$1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x + 1$$

$$0 + 1 \cdot x + 2 \text{ “excess } R.$$

By analogy with the coding method that uses the generator matrix, we obtain check symbols for the equality $R + P = 0$, where R is the division excess, is the vector of the check symbols $R = (012)$, $= (021)$.

Thus, we find the message code $U(x)=x^6+2 \cdot x^5+2 \cdot x^4+x^3+2 \cdot x+1$.

The operations of division are easier to perform by representing the codes in the forms of ratios in the respective positions of digits $m = (a_3, a_2, a_1, a_0)$. From the example $m(x) = 1221$.

After the operation of multiplication by x^3 (the three-digit shift to the left) $m(x) \cdot x^3 = 1221000$. The generator polynomial $q(x) = 1011$. The process of division. The check digit positions appear from the condition $R + P = 0$ ($P = 021$).

$$\begin{array}{r} 1221000 \\ 1011 \end{array} \left| \begin{array}{l} 1011 \\ 1211 \end{array} \right.$$

$$\begin{array}{l} 2100 \\ 2022 \end{array}$$

1110
 1011
 1020
 1011

012- “excess R.
 Check of correctness of coding by syndrome S.

1221.021	1011
1011	1211

2100
 2022
 1112
 1011
 1011
 1011

000- “syndrome S.

For each generator polynomial, we find syndromes of the type “1” and “2” errors. For the code (7,4) and the polynomial $q(x) = 1011$, 14 values of errors syndromes are provided in Table 1.

Table 1. Syndromes of errors for the triple polynomial 1011

Errors “1”	Errors “2”
$S_{11}=121$	$S_{21}=212$
$S_{12}=211$	$S_{22}=122$
$S_{13}=220$	$S_{23}=110$
$S_{14}=022$	$S_{24}=011$
$S_{15}=100$	$S_{25}=200$
$S_{16}=010$	$S_{26}=020$
$S_{17}=001$	$S_{27}=002$

The fact draws our attention that syndromes of errors in one digit position are reverse by (mod3), i.e. $S_{1j}+S_{2j}=0$.

The correcting capacity of the code is much higher with the base 3 than it is with the base 2.

For example, for the code (7,4), the binary code corrects 7 single errors, at that the set of codes of syndromes $2^{n-m}=2^3$ is used in full. Similar

estimations of the triple code show that 14 single errors are corrected. At that, the set of syndrome codes $2^{n-m}=3^3$ is used partially and 12 (27^{15}) syndrome codes can be used for correction of some double errors.

As the block size of the code (n, m) increases, the anti-jam properties of the triple code increase in geometric sequence if compared to the binary code.

The properties of the qualitative growth of anti-jam of the codes with larger base (2^d , where $d=2, 3, 4, \dots$) are implemented in the Reed–Solomon codes [3]. Development of analogs of the Reed–Solomon codes for bases 3^d produces effect that is even more impressive. The corrective capacity of the triple code in channel codecs is substantially higher than the one of the binary code for the same blocks (n, m) [14].

CONCLUSIONS

The specific feature of the triple code is the change of the algorithm of digit positions anti-jam formation. For channel decoding of the triple code, the methods of syndrome decoding are applicable.

REFERENCES

1. Kolmogorov, A.N., Three approaches to the definition of the concept “Quantity of information”. *Problemy Peredachi Informatsii*, 1965; **1**(1): 3-8.
2. Kharinov, M.V., Invariant triple encryption of information in digital image. *Trudy SPIRIAN*, 2006; **2**(3): 169-183.
3. Kuznetsov, V.S., Triple stage codes with CAM-9 modulation and their capabilities. *Elektrosvyaz*, 2009; **3**: 30-33.
4. Belyuchenko, I.M., Peculiarities of decryption of BI codes. *Elektrotekhnicheskyye i Informatsionnyye Kompleksy i Sistemy*, 2008; **4**(1-2): 33-38.
5. Belyuchenko, I.M., Variations of the triple code. *Elektrotekhnicheskyye i Informatsionnyye Kompleksy i Sistemy*, 2011; **7**(3): 17-20.
6. Artuschenko, V.M. and B.A. Kucherov, Analysis of information exchange in the process of distribution of control facilities for spacecrafts with resource restrictions. *European Science and Technology: In the Proceedings of the VII international research and practice conference*,

- Vol. II, Munich, December 27th – 28th, 2013. Germany, Munich: the publishing office Vela Verlag Waldkraiburg, 2013; 243-246.
7. Artuschenko, V.M. and B.A. Kucherov, Analysis of the possibilities of using spacecraft flight model for the distribution of funds management. In the Proceedings of the IX Mi' dzynarodowej naukowii-praktycznej konferencji «Perspektywiczne opracowania s nauk technikami-2013», Vol. 34. Nowoczesne informacyjne technologie. Przemyl, Nauka i studia, 2013; 26-30.
 8. Artyushenko, V.M. and V.I. Volovach, Statistical Characteristics of Envelope Outliers Duration of non-Gaussian Information Processes. In the Proceedings of the IEEE East-West Design & Test Symposium (EWDTS'2013), Rostov-on-Don, Russia, September 27–30, 2013. Kharkov: KNURE, 2013; 137-140.
 9. Artuschenko, V.M. and B.A. Kucherov, Evaluation of interference situation on board the spacecraft and earth stations in the corporate satellite communication systems. In the Proceedings of the X International scientific and practical conference «Trends of modern science». Vol. 26. Technical sciences. Sheffield, Science and education LTD, 2014; 65-67.
 10. Artuschenko, V.M. and B.A. Kucherov, 2014. Optimization of parameters of ground station of satellite communication system. European Science and Technology: In the Proceedings of the VII international research and practice conference, Vol. II, Munich, April 23th – 24th, Germany, Munich: the publishing office Vela Verlag Waldkraiburg, 2014; 397-400.
 11. Artuschenko, V.M. and B.A. Kucherov, Analisi della uso della tecnologia nella distribuzione dei controlli veicoli spaziali. Italian Science Review, 2014; **3**(12): 50-53.
 12. Abbasova, T.S., E.M. Abbasov and G.N. Isaeva, Conductivity testing communication lines for research noise-proof multiservice cable systems. European Science and Technology: In the Proceedings of the VII international research and practice conference, Vol.II, Munich, April 23th – 24th, 2014. Germany, Munich: the publishing office Vela Verlag Waldkraiburg, 2014; 390-393.
 13. Artyushenko, V.M. and T.S. Abbasova, Service of information systems in electrotechnical facilities. Monograph. Moscow: FGOUVPO RGUTiS, 2010; 98.
 14. Belyuchenko, I.M., Channel codecs of the triple code. Elektrotekhnicheskiye i Informatsionnyye Kompleksy i Sistemy, 2012; **8**(2): 30-33.