

Analysis Workflows Gear Hydraulic Machines

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The analysis workflow in a gear hydraulic machines of different designs. Coefficients of redistribution of moments from unbalanced hydrostatic forces. The dependences of the changes in the coefficients of redistribution of moments from the main parameters of the teeth.

Key words: Gear hydraulic, continuously variable transmission, differential hydra-mechanical variator, mechanical diagram, high-torque differential hydra-mechanical variator.

Due to the simplicity of the design, gear hydraulic machines are commonplace as non-regulated pumps used to supply non-adjustable pumps applied for small capacity hydraulic transmission with throttle control for lubrication supply and the systems control supply. Geared hydraulic transmission machines are the reversible mechanisms, being the simplest and having the lowest cost of all hydrostatic machines. However, they have failed to find a wide application in the transmission vehicles due to the complexity of continuous variable torque provision at the output shaft and its rotation speed change. The problem of the geared hydraulic machines regulation is solved by their converting into hydro-mechanical differential mechanisms¹.

Forces and torques in gear hydraulic machines

The use of the geared motors and hydraulic pumps as actuating mechanisms in the differential hydrostatic transmissions is based on their following properties^{2,3}:

- Geared hydraulic machines can easily be transformed into a differential mechanism, where the central gear is leading in case of the hydraulic pump, or driven in case of the hydraulic motor; the gears scoring at least two and installed on the axes in a moving body are the satellites;
- The flow rate value on each of the gears varies with the size of the gears and, consequently, with the change of the gear transmission ratio there between;
- The flow rate addition occurs partly due to the forces in the toothed wheel gearing interaction by power transmission from one gear to another, and partly due to overcoming of the resistance moments caused by the action of unbalanced hydrostatic forces of fluid pressure, which create unequal circumferential moments on each of the gears.

Moments applied to the gears of the gearing hydraulic machine are determined by the effect of the fluid pressure forces to the same areas, which determine the supply process formation. To calculate the supply presented at the Figure 1 the

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hydraulic pump gears are replaced by flat systems, in which the point A is a gearing point at a given moment of time t . The direct line $O_1A = \rho_1$ and $O_2A = \rho_2$ connecting this point with the gear center, and the direct O_1F_1 and O_2F_2 separate the suction and the injection areas.

By a uniform rotation of the gears, the power imparted to the working fluid:

$$p_u dV = (p_2 - p_1) dV \quad \dots(1)$$

where dV – the volume pumped into the pressure line during the time dt ; p_1, p_2 – suction pressure and discharge respectively, MPa; p_i – pump pressure, MPa.

This energy is brought to the liquid as moments M_1 and M_2 , supplied from the driving shaft to the gears for overcoming of loads, which occur on the gears due to unbalanced hydrostatic forces. Neglecting the losses, the balance of power on two gears shall be as follows:

$$\dots(2)$$

where M_1, M_2 – moments of resistance on the master and the slave wheels, N·m; ω_1, ω_2 – angular velocity of rotation of driving and driven wheels; φ_1, φ_2 – be the angles of rotation of the respective gears.

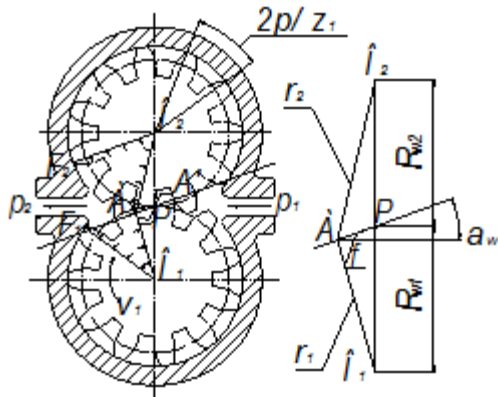


Fig. 1. Kinematic scheme for calculation of the feed pump with one driven wheel

The moments I_1 and I_2 shall be determined by the following formulas:

$$\dots(3)$$

$$M_2 = p_u b \frac{R_{a2}^2 - \rho_2^2}{2} \quad \dots(4)$$

where b – the width of the gears, m; M_1, M_2 – the radii of the circumferences of the tops of the teeth, m.

The mean value of torque on the diving gear is calculated for them, as for other hydraulic machines, by the formula:

$$M_u = \frac{V_o p_u}{2\pi} \quad \dots(5)$$

where V_o – the working volume of the hydraulic machine, m^3 .

Bearing in mind that the pump supply presents a volume change in a time period, we get the following:

$$Q = \frac{dV}{dt} = \frac{M_1 \omega_1 + M_2 \omega_2}{p_u} = \frac{b \omega_1}{2} (R_{a1}^2 + R_{a2}^2 \frac{\omega_2}{\omega_1} - \rho_1^2 - \rho_2^2 \frac{\omega_2}{\omega_1}) \dots(6)$$

The distances ρ_1 and ρ_2 can be determined from Figure 1 by the cosine law.

$$\rho_1^2 = R_{w1}^2 + f^2 - 2R_{w1}f \cos(90 - \alpha_w) \dots(7)$$

$$\rho_2^2 = R_{w2}^2 + f^2 - 2R_{w2}f \cos(90 + \alpha_w) \dots(8)$$

where R_{w1}, R_{w2} – the radii of the pitch circles of the gears, m; f – the distance from the hooking points to the pole, m; α_w – pressure angle, °.

The final formula for determining the pump flow shall be written as follows:

$$Q = \frac{b \omega_1}{2} \left[K - f^2 \left(1 + \frac{R_{w1}}{R_{w2}} \right) \right] \quad \dots(9)$$

where

$$K = \left(R_{a1}^2 + R_{a2}^2 \frac{R_{w1}}{R_{w2}} - R_{w1}^2 - R_{w1} R_{w2} \right)$$

– coefficient depending only on the geometry of the gears, m^2 .

Instantaneous torque value on the shaft of the hydro-pump driven by one external gearing wheel can be determined from the energy balance. Having in mind, that on the one part the pump

supply flow $Q = \frac{dV}{dt} = \frac{M_1 \omega_1 + M_2 \omega_2}{p_u}$, and on the other part $V = Q / n_1$, we shall obtain the following:

$$M = \frac{p_n Q}{2\pi n_1} = \frac{p_n Q}{\omega_1} = \frac{M_1 \omega_1 + M_2 \omega_2}{\omega_1} = M_1 + M_2 \frac{\omega_2}{\omega_1} \dots(10)$$

That means, part of the flow rate is taken up directly on the driving gear (on the hydraulic pump shaft), and the other part - on the driven wheel of the hydraulic pump, and is transmitted through the gear ratio from the driven wheel to the hydraulic pump shaft. The moments change schedule from the output rotation angle of the pump shaft is shown in Figure 2.

Taking into the account the formula (9), the expression (10) shall be written as:

$$M = \frac{p_n b}{2} \left[K - f^2 \left(1 + \frac{R_{w1}}{R_{w2}} \right) \right] \dots(11)$$

One duty cycle of the machine corresponds to the gears rotation to the angular pitch of $2\pi/z_1$ (z_1 – a number of teeth of the pinion). The point of teeth contact is moved along the line of engagement. This causes a variability of supply, and consequently – of the torque during the working cycle^{4,5}.

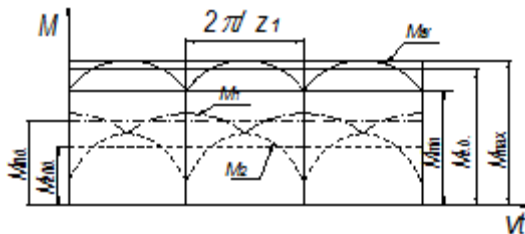


Fig. 2: Points in the hydraulic pump with one drive wheel

It is known from the theory of gearing that at turning within the angular pitch the segments ρ_1 and ρ_2 length varies according to a parabolic law. Geometric values characterizing the gearing enable to express the dependence of the hydraulic machine moment on the gear angle.

The maximum value M_{max} is at $f=0$:

$$M_{max} = \frac{p_n b}{2} K \dots(12)$$

The minimum value M_{min} at $f = p_{bn}/2$ ($p_{bn} = \pi m \cos \alpha_w$ – the main gearing step).

$$M_{min} = \frac{p_n b}{2} \left[K - \frac{p_{bn}^2}{4} \left(1 + \frac{R_{w1}}{R_{w2}} \right) \right] \dots(13)$$

At quadratic law of the moment change, the values \dot{I}_{max} and \dot{I}_{min} enable to determine the mean value of the moment $\dot{I}_{e.o}$.

$$\dots(14)$$

The average (theoretical) values of the resistance moment M_1 and M_2 can be expressed in terms of the driving and driven gears supply respectively, at turning to an angle step.

$$M_{1cp} = \frac{p_n Q_{1cp}}{\omega_1} \dots(15)$$

$$M_{2cp} = \frac{p_n Q_{2cp}}{\omega_2} \dots(16)$$

$$Q_{1cp} = V_{1o\ddot{a}\ddot{a}} z_1 n_1 \dots(17)$$

$$\dots(18)$$

where $V_{1o\ddot{a}\ddot{a}}$ – volume of working liquid delivered by the gear pump leads when you turn on the angular step $2\delta/z_1$; $V_{2o\ddot{a}\ddot{a}}$ – volume of the working fluid supplied by the hydraulic pump driven wheel when turning at the angular step $2\delta/z_2$.

;

$$\dots(19)$$

$$V_{2o\ddot{a}\ddot{a}} = \int_{-f_2}^{f_1} dV_2 = \frac{b}{2R_{b2}} \int_{-f_2}^{f_1} [R_{a2}^2 - R_{w2}^2 - f^2 + 2R_{w2} f \cos(90 + \alpha_w)] df \dots(20)$$

According to the theory of gearing

$$\frac{df}{dt} = \omega R_b \dots(21)$$

where R_b – the radius of the basic circle of the gears, m.

$$R_b = \frac{mz}{2} \cos \alpha_w, \dots(22)$$

where m – module gear, m.

For the case, when the engagement length

is equal to the unit ($\dot{a} = 1$), $f_1 = f_2 = \frac{p_{bn}}{2}$,

$$V_{1o\ddot{a}\ddot{a}} = \frac{b}{2R_{b1}} \left[(R_{a1}^2 - R_{w1}^2) p_{bn} - \frac{p_{bn}^3}{12} + \frac{p_{bn}^2}{2} R_{w1} \cos(90 - \alpha_w) \right]$$

$$V_{2max} = \frac{b}{2R_{b2}} \left[(R_{a2}^2 - R_{w2}^2) p_{bn} - \frac{p_{bn}^3}{12} + \frac{p_{bn}^2}{2} R_{w2} \cos(90 + \alpha_w) \right]$$

and taking into account the equation (22), the expressions for the theoretical moments of resistance can finally be written as follows:

$$M_{1cp} = \frac{p_n b}{2} \left[R_{a1}^2 - R_{w1}^2 - \frac{p_{bn}^2}{12} + \frac{p_{bn}}{2} R_{w1} \cos(90 - \alpha_w) \right]; \quad \dots(23)$$

$$M_{2cp} = \frac{p_n b}{2} \left[R_{a2}^2 - R_{w2}^2 - \frac{p_{bn}^2}{12} + \frac{p_{bn}}{2} R_{w2} \cos(90 + \alpha_w) \right]. \quad \dots(24)$$

The ratio of the resistance moments in the driving and driven gears can be presented through the coefficient λ_p , which is defined as

$$\lambda_H = \frac{M_{1cp}}{M_{2cp}} = \frac{z_1 [12 + 3\pi \cos \alpha_w \cos(90 - \alpha_w)] + 12 - \pi^2 \cos^2 \alpha_w}{z_1 i_{12} [12 + 3\pi \cos \alpha_w \cos(90 + \alpha_w)] + 12 - \pi^2 \cos^2 \alpha_w}. \quad \dots(25)$$

The dependence of the coefficient λ_H of the hydro-pump with external gear engagement on the gearing parameters are shown in Figure 3.

The conclusion following the formula (25) and the schedule analysis can be that the greatest impact on the coefficient λ_H is rendered by the transmission ratio of the hydro-pump value and the gearing angle α_w . The increase in the number of driven wheels of the pump shall not affect the redistribution of moments [6].

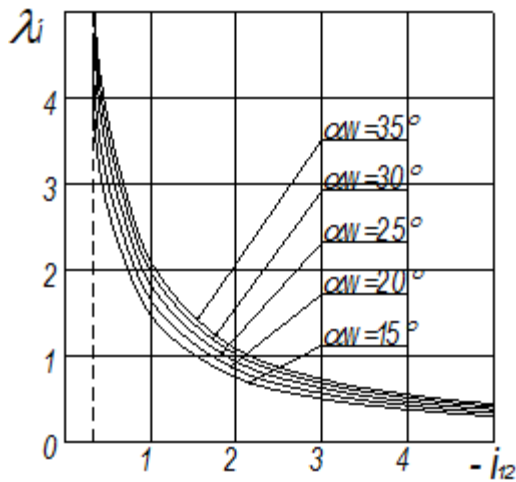


Fig. 3. Dependence of the coefficient of λ_H from the gear ratio of the hydraulic pump i_{12} and the angle gear α_w

The hydraulic pumps with internal gearing also manifest a redistribution of moment between the driving and driven gears, but the value of the coefficient λ_H is different; it also depends on either the outer or the inner wheel is driving.

We shall consider the Figure 4 to calculate the resistance moments in hydraulic pumps with an internal driving wheel, where the gears of the hydro-pump are replaced by flat systems, where the point A – is a gearing point at a given moment of time t . Direct lines $O_1A = \rho_1$ and $BA = \rho_2$, as well as the direct lines O_1F_1 and B_2F_2 , separate areas of suction and injection^{7,8}.

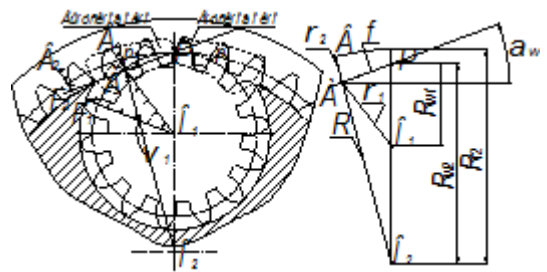


Fig. 4. The scheme for calculating the feed pump with the internal gear

The moments M_1 and M_2 are determined by the following formulas:

$$M_1 = p_n b \frac{R_{a1}^2 - \rho_1^2}{2} \quad \dots(26)$$

$$M_2 = p_n b \frac{R_{f2}^2 - R_{a2}^2 - 2R\rho_2 - \rho_2^2}{2} \quad \dots(27)$$

where R_{f2} – radii of the circles of teeth cavities crown wheel, m.

Distances ρ_1, ρ_2 and R can be determined from the Figure 4.

$$\rho_1^2 = R_{w1}^2 + f^2 - 2R_{w1}f \cos(90 - \alpha_w) \quad \dots(28)$$

$$\rho_2^2 = (R_{f2} - R)^2 \quad \dots(29)$$

$$R^2 = R_{w2}^2 + f^2 - 2R_{w2}f \cos(90 - \alpha_w) \quad \dots(30)$$

Similarly to the pump with the external engagement gears, the moments M_{1cp} and M_{2cp} are determined through the driving and driven gears supply respectively at turning to an angle step by the formulas:

$$M_{1cp} = \frac{p_n b}{2} \left[R_{a1}^2 - R_{w1}^2 - \frac{p_{bn}^2}{12} + \frac{p_{bn}}{2} R_{w1} \cos(90 - \alpha_w) \right]; \quad \dots(31)$$

$$M_{2cp} = \frac{p_n b}{2} \left[R_{a2}^2 - R_{w2}^2 + \frac{p_{bn}^2}{12} - \frac{p_{bn}}{2} R_{w2} \cos(90 - \alpha_w) \right]; \quad \dots(32)$$

and the coefficient λ_H :

$$\lambda_H = \frac{M_{1cp}}{M_{2cp}} = \frac{z_1 [12 + 3\pi \cos \alpha_w \cos(90 - \alpha_w)] + 12 - \pi^2 \cos^2 \alpha_w}{z_1 i_{12} [12 - 3\pi \cos \alpha_w \cos(90 - \alpha_w)] - 12 + \pi^2 \cos^2 \alpha_w} \quad \dots(33)$$

The coefficient λ_H of the pump with internal gearing dependence on the parameters of the gearing engagement with an internal driving wheel is shown in Figure 5.

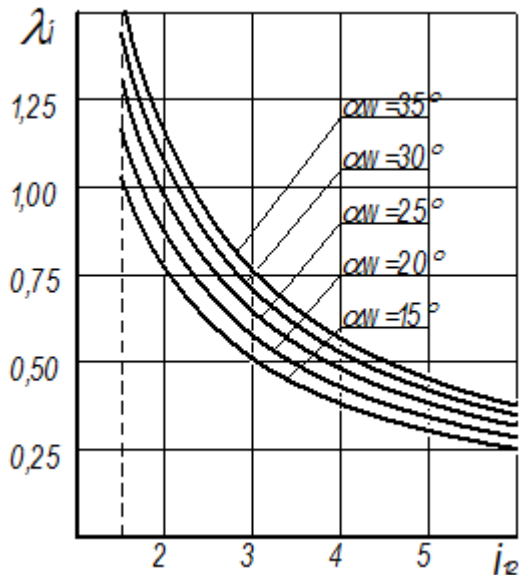


Fig. 5. Dependence of the coefficient of λ_H from the ratio of pump i_{12} and the angle α_w at internal drive gear

Let us consider the pump with internal gearing and with a drive to the crown wheel (Figure 6). In this case, the values of the moments \dot{I}_1 and \dot{I}_2 are determined by the following formulas:

$$M_1 = p_n b \frac{R_{f1}^2 - R_{a1}^2 - 2R\rho_1 - \rho_1^2}{2} \quad \dots(34)$$

$$M_2 = p_n b \frac{R_{a2}^2 - \rho_2^2}{2}, \quad \dots(35)$$

where R_{f1} – the radii of circles of the tooth of the crown wheel, m.

Distances ρ_1, ρ_2 and R can be determined from the Figure 5.

$$\rho_1^2 = (R_{f1} - R)^2; \quad \dots(36)$$

$$R^2 = R_{w1}^2 + f^2 - 2R_{w1}f \cos(90 + \alpha_w); \quad \dots(37)$$

$$\rho_2^2 = R_{w2}^2 + f^2 - 2R_{w2}f \cos(90 + \alpha_w) \quad \dots(38)$$

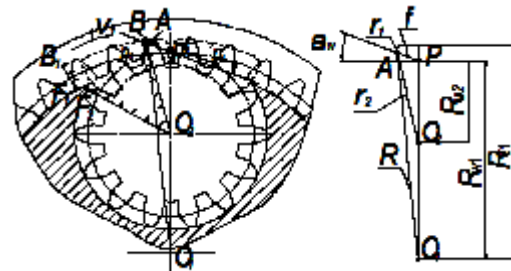


Fig. 6. Scheme for calculation of the flow hydraulic pump with gears internal gears

In this case, the moments of resistance are defined as

$$M_{1cp} = \frac{p_n b}{2} \left[R_{w1}^2 - R_{a1}^2 + \frac{p_{bn}^2}{12} - \frac{p_{bn}}{2} R_{w1} \cos(90 + \alpha_w) \right]; \quad \dots(39)$$

$$M_{2cp} = \frac{p_n b}{2} \left[R_{a2}^2 - R_{w2}^2 - \frac{p_{bn}^2}{12} + \frac{p_{bn}}{2} R_{w2} \cos(90 + \alpha_w) \right]; \quad \dots(40)$$

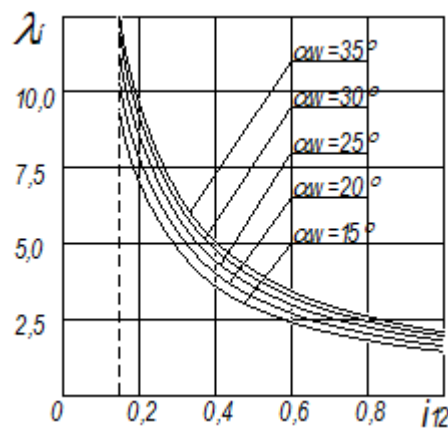


Fig. 7. Dependence of the coefficient of λ_H from the ratio of pump i_{12} and the angle α_w at an outside pinion

and the coefficient λ_H :

$$\lambda_H = \frac{M_{1cp}}{M_{2cp}} = \frac{z_1[12 + 3\pi \cos \alpha_w \cos(90 + \alpha_w)] + 12 - \pi^2 \cos^2 \alpha_w}{z_1 i_{12}[12 - 3\pi \cos \alpha_w \cos(90 + \alpha_w)] - 12 + \pi^2 \cos^2 \alpha_w} \dots(41)$$

Dependences of the hydro-pump with the internal gearing coefficient $\dot{\epsilon}_i$ on the parameters of the gearing with the outer driving wheel are shown in Figure 7.

CONCLUSION

1. The analysis of the forces and moments acting in the gear hydraulic machines enable to make a conclusion that the gear-type hydraulic machines can be converted into the hydro-mechanical differential mechanisms that can be regarded as the first step of hydro-mechanical variators.

2. Hydro-mechanical differential mechanism has the following features: - Two degrees of freedom; - The moments distribution coefficient from the unbalanced hydrostatic pressure forces determines the presence and magnitude of holonomic and non-holonomic constraints between the driving and driven units of hydraulic gear machines; - The use of working fluid flow as a hydraulic connection between hydro-mechanical differential and the one of the possible mechanisms of hydraulic energy converting into mechanical one, which enables to create a continuously variable hydro-mechanical transmission; - Mechanical moment, taken from the carrier of the hydro-mechanical differential mechanism can be added up to another mechanical moment acquired upon the conversion of the hydraulic flow rate; - At changing of the angular velocities of hydro-mechanical differential mechanism units, there occurs a change in hydro-pump supply, which provides an internal automatic performance at converting of hydraulic flow rate into a mechanical one.

This will enable to solve the problem of high-moment hydro-mechanical variators development.

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