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The Method of Organizing the Iterative Process of the System of the Linear Algebraic Equations Solution Excluding the Multidigit Multiplication Operation

Pavel Pavlovich Kravchenko and Liubov Vladimirovna Pirskaya

Southern Federal University, Russia, 344006, Rostov-on-Don, Bolshaya Sadovaya str., 105/42,

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An iterative method of the systems of the linear algebraic equations solution, excluding the multidigit multiplication operation while designing the special-purpose computing facilities is considered. Algorithmization of the iteration process is based on the use of the first order delta-modulations with the variable quantum that can significantly reduce the number of iterations in comparison with the using of the constant quantum. The main theoretical principles justifying the approximate solution of the problem of minimizing the number of iterations when using the variable quantum are highlighted for the first time. The optimal estimations characterizing the duration of idealized iterative loops with constant quantum of a certain value are formed in theoretical justification quasi-optimal conditions determining a way of the representation a sequence of variable quanta values in the loops and based on using two or four idealized iterations within each loop are developed for the purpose of the real processes. Besides the quanta values shall be represented in the form of 2^{-S}, $S \in N$, that allows at each iteration submitting the multiplication of the matrix coefficient by the quantum represent in the form of a shift operation by S binary digits. The introduction of this way of representing the quantum allows realizing the implementation of the iterative process without using the multiplication of the multi-digit codes. There are also effective ways to complete the iteration in each loop, allowing, in particular, to reduce the number of iterations in the loop per unit. The findings of investigation of the iterative solution of various systems of the linear algebraic equations different with convergence rate are given in the work. The possibility of reduction the number of iterations in comparison with the using the delta-modulation with the constant quantum to hundreds – thousands of times is shown when ensuring the identical accuracy. When performing the experiments the attention to the manifestation of the effective usage of the first proposed method of four idealized iterations in the loop is paid. The number of iterations with variable quantum is to a considerable extent approximate the number of iterations to the simple iteration method.

Key words: Iteration methods, systems of the linear algebraic equations, the first order delta-modulation, special-purpose control unit.

The creation of the high-performance computing systems operating in real time, has been associated, in particular, with the use of the task-oriented specialized tools based on FPLD (Field Programmable Logic Device), CIS (Combat Information System), controllers, microprocessors (Maxfield, 2007). The features of algorithmic maintenance allowing to reach, whenever possible, the highest quality performance index at the minimized expenses of resources (equipment) are of great importance in the problem solving of the special-purpose computing facilities effective design (Yang, & Ziavras, 2005; Zhang, *et al*., 2008; Zhang, *et al*., 2012).

As separate component of the tasks that

* To whom all correspondence should be addressed.

can be accomplished within the framework of the special-purpose computing facilities, there are often the targets of computing mathematics, in particular in the form of systems of the linear algebraic equations (linear systems). When using the iterative methods of the solution of these systems there is the need to use many multipliers, the realization of which requires significant hardware resources and the machine time cost at work with full codes of the variables and coefficients.

In connection with the above-noted the creation of such algorithmic transformations of the iterative processes for linear systems solving, which are focused on improving the qualitative characteristics of the corresponding computational structures: a reduction of the necessary equipment and ensuring the number of iterations, approximated to the fixed point iteration method. There are known the iterative methods of linear systems solving, excluding a multiplication based on the first and second order delta-modulation with constant and variable quantum (Malinovsky, *et al*., 1977; Boyong, *et al*., 1977; Tretyakov, 1978; Tretiakov, 1978; Nikolaev 1968; Klochkov, & Nikolaev, 1962; Kravchenko, 1983; Kravchenko, 1989; Gomozov, 2009; Gomozov, & Ladyzhensky, 2010, Ledyankin, 1979). The known disadvantage of the methods with the constant quantum here is a great many iterations. The use of the variable quantum allows reducing the number of iterations. The essence of the methods for linear systems solving on the basis of the first order delta-modulations with the variable quantum is in representation of the active task in the form of the difference equation system of the first order; the formation of the variables is carried out on the basis of the constants according to the module (2^{-s}, $s \in N$) and differing in the sign values of the first differences (transformation quanta), and the multiplication operation is replaced by the operation, the main components of which are the shift and multiplication by ± 1.

Almost all the works concerned the algorithmic organization of the iterative process for linear systems solving with the use of the first order delta-modulation with the variable quantum has not got any information about the theoretical justification for the selection of the best relation between the quanta of the adjacent loops; on the one hand the ways to specify these relations are introduced heuristically, on the other hand they are not fully cover the theoretical and practical interest of these relations (Malinovsky, *et al*., 1977; Boyong, *et al*., 1977; Tretyakov, 1978; Tretiakov, 1978; Nikolaev, 1968; Klochkov, & Nikolaev, 1962; Kravchenko, 1983; Kravchenko, 1989; Gomozov, 2009; Gomozov, & Ladyzhensky, 2010).

In addition, the following observations should be noted. In the works (Malinovsky, *et al*., 1977; Boyong, *et al*., 1977; Tretyakov, 1978; Tretiakov, 1978; Gomozov, 2009; Gomozov, & Ladyzhensky, 2010) the proposed algorithms are characterized by high computational complexity when implementing the iteration process due to the necessity of performing the squaring of the discrepancies across all equations at each iteration, summing them and highlight the smallest of the current according to the amount of values across the iterations. The squaring of the discrepancies is associated with the need to use the multidigit multipliers, which is contrary to the original goal – implementation of the special-purpose control units of linear systems solution with the possible of the exception of such devices. The iterations termination moment is fixed either with the use of a certain constant, the determining value of which is uncertain, or according to the number of iterations that is also a problem of the preliminary estimate.

In the work (Kravchenko, 1983) an estimate of the number of iterations at the intervals of the consistent transformation quantum (loops) involves using the rules of the matrix of coefficients and is estimated to be in the “worst impacts”, which generally causes the device to operate at a higher number of iterations with the slight chance of the worst impacts manifestation. The above-mentioned estimate is applicable in cases, if the norm of the matrix of coefficients is less than unity; but at the same time, the iterative processes under certain conditions can be successfully implemented at a rate of greater than unity. The process of linear systems solution must be preceded by the number of iterations in the loop, using, in particular, the division operation that under the conditions of the perform these calculations with the help of the specialized computing devices is not appropriate.

In the work (Ledyankin, 1979), the computer process organization is based on the cost of hardware and time resources of the variable increments; simultaneous operation with equivalent values of increments of the modules creates a problem in the implementation of the synchronous performance and cost effective hardware parallel operation device of a calculator. The main theoretical principles justifying the approximate solution of the problem of minimizing the number of iterations when using the variable quantum are highlighted for the first time. The possible expansion of the recommended relation between the quanta of the adjacent loops is substantiated, also effective ways of completion of iterations in each loops are offered. Some results of the carried out experiments are given.

METHODS

The problem statement

Let the original linear system, containing a matrix of constant coefficients with regular free members, is given by:

$$\mathrm{B}\mathrm{Y}^*(t) = G \qquad ...(1)$$

We shall convert the system:

$$\mathrm{Y}^*(t) = \mathrm{A}\mathrm{Y}^*(t) + D$$

We pass to a record form with use of an iterative method and residual introduction *z(t)*:

$$z(t) = \mathrm{Y}(t) - \mathrm{A}\mathrm{Y}(t) - D \qquad ...(2)$$

In these systems $\mathrm{B} = [b_{rj}]$, $\mathrm{A} = [a_{rj}]$ – the coefficient matrix of the dimension $n \times n$; , $\mathrm{Y}^*(t)$ – column vectors, respectively, the right parts and unknown systems; $z(t)$, $\mathrm{Y}(t)$ – column vector of discrepancies and approximate values of the unknowns; t – independent variable; $\det A \neq 0$.

Algorithm for the approximate solution of (1) with the use of the first order delta-modulation and the variable quantum will be presented in the following differential form for the *i*-th step, at the initial conditions $Y_{r01} = 0$, $z_{r01} = -D_r$, $r = \overline{1,n}$, (Kravchenko, 1983; Kravchenko, 2008):

- the formation of the quanta characters of the first differences of the variables:

$$\Delta_{ril} = -sign(z_{r(i-1)l}); \ \Delta_{ril} \in \{+1,-1\}; \ r = \overline{1,n},$$

$i = 1, 2, \ldots, R_l$; $l = 1, 2, \ldots, P$;

- demodulation:

$$\nabla Y_{ril} = c_l^* \Delta_{ril}; \ Y_{ril} = Y_{r(i-1)l} + \nabla Y_{ril};$$

- the formation of the first difference of the transformed variable:

$$\nabla y_{ril} = \sum_{j=1}^{n} a_{rj} c_l^* \Delta_{jil};$$

- residual meaning creation:

$$\nabla z_{ril} = \nabla Y_{ril} - \nabla y_{ril};$$

$$z_{ril} = z_{r(i-1)l} + \nabla z_{ril}.$$

Let us denote the algorithm (3). In the algorithm (3) c_l – module weight of the transformation quantum in l-th loop ($c_l > 0$), *P* is a number of iterative loops at constant on the module of the quanta values, R_l – number of iterations in the loop. In addition, for the joints of the adjacent loops the following correlations are fair: $Y_{r0l} = Y_{rR(l-1)}$; $z_{r0l} = z_{rR(l-1)}$.

It is required to find conditions of the optimized choosing of the numbers of the loops and the quantum values in the loops that allow optimizing the number of iterations which are characterized by convergence of the iterative process of linear systems solution.

Original position in solving the optimization problem

From a consideration of the algorithm (3) can be written for *l*-th loop.

$$z_{ril} = z_{r(i-1)l} - c_l^* sign(z_{r(i-1)l}) - \nabla y_{ril}.$$

Suppose that $\nabla y_{ril} = 0$. When performing this condition at the *i*-th step of the *r*-th equations the module $z_{r(i-1)l}$ will decrease, i.e this $|z_{ril}| < |z_{r(i-1)l}|$ will be at a size c_l^*. The residual $z_{r(i-1)l}$, $i = 1, 2, ..., R_l$ decreases, and its decreasing value with increasing *i* necessarily passes through zero, in particular, changing the sign. Thus the quantity of steps (iterations) in the present circumstances before entering into zero for $l = 1$ will make

$$R_1 = \frac{|z_{r01}|}{c_1}.$$

After passing through zero the maximum residual value in the present circumstances

$$z_{rR1} = z_{r02},\ |z_{r02}| \le c_1,$$

provided that $sign(z_{r02}) = -sign(z_{r01})$ or $z_{r02} = 0$. Further, the ratio $z_{r02} = 0$ will be considered only in terms of the iteration completion. Respectively for the l-th, $l = 1, 2, \ldots, P$ value R_l can be written:

$$R_l = \frac{|z_{r0l}|}{c_l}. \qquad \text{...(4)}$$

As a condition to completion of the transition process in the loop for further research now we shall accept

$$|z_{rRl}| \le c_l,\ l = 1, 2, \ldots, P,\ r = \overline{1, n}. \qquad \text{...(5)}$$

On the basis of the above, we can make the following conclusions:

1. Provided $\nabla y_{ril} = 0$ the residual meaning at the end of the iteration process changes the sign (or equal zero).
2. The number of iterations in the loop can be evaluated using the expression (4).

When performing real-converging iterative process, the probability values $\nabla y_{ril} = 0$ is small, and this consideration must be taken into account $\nabla y_{ril} \ne 0$. If $sign(\nabla y_{ril}) = sign(z_{r(i-1)l})$, namely

$$z_{ril} = z_{r(i-1)l} - c_l^* sign(z_{r(i-1)l}) - |\nabla y_{ril}| sign(z_{r(i-1)l})$$

and

$$z_{ril} = z_{r(i-1)l} - (c_l^* + |\nabla y_{ril}|) sign(z_{r(i-1)l}),$$

then the residual reduction rate increases (number of iterations in the loop decreases) through the appropriate reduction of residual at each step by an amount exceeding the quantum in the expression $(c_l^* + |\nabla y_{ril}|)$.

If $sign(\nabla y_{ril}) = -sign(z_{r(i-1)l})$, namely

$$z_{ril} = z_{r(i-1)l} - c_l^* sign(z_{r(i-1)l}) + |\nabla y_{ril}| sign(z_{r(i-1)l})$$

and

$$z_{ril} = z_{r(i-1)l} - (c_l^* - |\nabla y_{ril}|) sign(z_{r(i-1)l}),$$

then the residual reduction rate decreases (number of iterations in the loop increases) through the appropriate reduction of residual at each step by an amount exceeding the quantum in the expression $(c_l^* - |\nabla y_{ril}|)$; in this case the condition for the residual reduction is $c_l^* > |\nabla y_{ril}|$.

In this regard, we can draw the following conclusions:

$z_{rRl} = 0$ 1. The number of iterations in the loop with $\nabla y_{ril} \ne 0$ may be larger or less than (4).

2. When converging computing process the value residual at the end of the loop reverses the sign (or equals zero), i.e.

$$sign(z_{rRl}) = -sign(z_{r(R-1)l}) \text{ or } , \qquad \text{...(6)}$$

$$l = 1, 2, \ldots, P,\ r = \overline{1, n}.$$

Summing up the foregoing, for the theoretical justification for conditions of the

Table 1. The results of the experiments

The method of organizing the iterative process of the linear systems solution		The number of iterations (24)	(25)	(26)	(27)
On the basis of the first order delta-modulations with the constant quantum		21064	19660	17613	19161
On the basis of the first order delta-modulations with the variable quantum	$R_{int} = 2$	15	19	22	51
	$R_{int} = 4$	18	28	31	43
	$R_{int} = 8$	29	56	51	58
Simple iteration		4	8	36	59
Presented in this paper (Kravchenko, 1983) (the algorithm works for the norm of the matrix coefficient, smaller than a one)		24	142	-	-

organization of the efficient iterative computing process with variable quantum of the first order delta-modulation we have:

1. As the main settlement on the correlation between the number of iterations in an idealized loop we shall take (4).
2. In the framework of the formation of the iterations termination moment in l -th loop we shall accept the condition (6), the corresponding to the time (increment number), to which along all linear system equations the signs of changing the residual sign are formed in this loop, i.e. completion of the iterative process.

The optimum estimation providing a minimum duration of the iterative processes when $\nabla y_{ril} = 0$

We shall look for assessment of the theoretical minimum number of iterations of the idealized process (number of steps of the delta-modulation of the testing the initial discrepancies when = 0). The number of iterations (the transient time) *M* of an idealized solution of linear system will present in the form of:

$$M = \sum_{l=1}^{P}, \; l = 1, 2, \ldots, P \qquad \text{...(7)}$$

or

$$M = \sum_{l=1}^{P} \frac{|z_{0l}|_{\max}}{c_l} \qquad \text{...(8)}$$

where $|z_{0l}|_{\max} = \max |z_{r0l}|, \; r = \overline{1, n}$.

Taking into consideration (5) for residual meaning we shall write down the limit correlation:

$$|z_{0l}|_{\max} = c_{l-1}, \; l = 1, 2, \ldots, P.$$

Expression (8) takes the form:

$$M = \frac{|z_{01}|_{\max}}{c_1} + \frac{c_1}{c_2} + \frac{c_2}{c_3} + \ldots + \frac{c_{P-1}}{c_P} \qquad \text{...(9)}$$

Accept for all loops of the same size:

$$R_l = R, \; l = 1, 2, \ldots, P. \qquad \text{...(10)}$$

The initial residual value $|z_{01}|_{\max}$ and weight of the minimum quantum c_p associated with the necessary of the accuracy requirement of the calculations, are set as the initial conditions. Taking into consideration the condition (9), (10) we get:

$$R = \sqrt[P]{\frac{|z_{01}|_{\max}}{c_p}} \qquad \text{...(11)}$$

In accordance with the expression (11) an assessment of the transient time (7) takes the form:

$$M = P \cdot \sqrt[P]{\frac{|z_{01}|_{\max}}{c_p}} \qquad \text{...(12)}$$

We shall continue to seek the conditions for ensuring the minimum value of *M*. Differentiate the expression (12):

$$M' = \left(P \cdot \sqrt[P]{\frac{|z_{01}|_{\max}}{c_p}}\right)' = P' \cdot \left(\frac{|z_{01}|_{\max}}{c_p}\right)^{\frac{1}{P}} + P \cdot \left(\left(\frac{|z_{01}|_{\max}}{c_p}\right)^{\frac{1}{P}}\right)' =,$$

$$= \left(\frac{|z_{01}|_{\max}}{c_p}\right)^{\frac{1}{P}} + P \cdot \left(\frac{|z_{01}|_{\max}}{c_p}\right)^{\frac{1}{P}} \cdot \ln\left(\frac{|z_{01}|_{\max}}{c_p}\right) \cdot \left(\frac{1}{P}\right)'$$

equate to zero

$$M' = \left(\frac{|z_{01}|_{\max}}{c_p}\right)^{\frac{1}{P}} - \left(\frac{1}{P}\right) \cdot \left(\frac{|z_{01}|_{\max}}{c_p}\right)^{\frac{1}{P}} \cdot \ln\left(\frac{|z_{01}|_{\max}}{c_p}\right) = 0$$

or

$$1 - \left(\frac{1}{P}\right) \cdot \ln\left(\frac{|z_{01}|_{\max}}{c_p}\right) = 0$$

and find the value of *P*:

$$P = \ln \frac{|z_{01}|_{\max}}{c_p} \qquad \text{...(13)}$$

As a result, the estimate of the number of iterations (the transient time) *M* subject to (11) and (13) takes the form:

$$M = \ln \frac{|z_{01}|_{\max}}{c_p} \cdot \left(\frac{|z_{01}|_{\max}}{c_p}\right)^{\frac{1}{\ln \frac{|z_{01}|_{\max}}{c_p}}} \qquad \text{...(14)}$$

Using the definition of the natural logarithm, to the expression (13) we shall write:

$$e^P = \frac{|z_{01}|_{\max}}{c_p} \qquad \text{...(15)}$$

At the same time, the expression (11) can be written as:

$$R^P = \frac{|z_{01}|_{max}}{c_P} \qquad ...(16)$$

From the equality of the right parts (15), (16) follows the equality of the left parts: $e^P = R^P$ from which for the optimal value we have *R* values:

$$R = e = 2{,}718281828.$$

Thus, when $\nabla y_{ril} = 0$ the optimal number of iterations idealized process in the loop that remains unchanged the quantum transformation, essentially, does not depend on the initial residual value $|z_{01}|_{\max}$ and on the weight of the minimum quantum c_p, and is a constant that is equal to Euler's number.

The formation of quasi-optimal parameter values for the real process

The essence of the iterative process of linear system solution without the use of the multiply multidigits numbers is that the value of permanent units (quanta) C_l, $l = 1, 2, ..., P$ shall be submitted in the form 2^{-s}, $s \in N$, and *P* and *R* – in the form of integer values P_{int} and R_{int}, respectively. With this algorithm (3) in each iteration the multidigit multiplication operation of the matrix coefficient by the transformation quantum can be represented by an shift operation to this coefficient on S binary digits with assigning the appropriate sign.

The relationship between the quanta of the adjacent loops can be written as:

$$c_{l-1} = c_l \cdot R,\ l = \overline{P,1}.$$

In order that all c_l, $l = 1, 2,..., P$ can be represented as 2^{-s}, $s \in N$, obviously, it is enough that $c_p = 2^{-s}$, $s \in N$ and

$$R_{int} = 2^k,\ k \in N \qquad ...(17)$$

As the integer values it is advisable to take the number closest to the optimum R = *e*, i.e. $R_{int} = 2$ and $R_{int} = 4$.

Next we define P_{int}. Let us take the logarithm of the expression (16):

$$\ln R^P = \ln \frac{|z_{01}|_{max}}{c_P}$$

Using the logarithm properties, we get:

$$P = \frac{\ln \dfrac{|z_{01}|_{max}}{c_P}}{\ln R}$$

For integer values P_{int}:

$$P_{int} = \left\lceil \frac{\ln \dfrac{|z_{01}|_{max}}{c_P}}{\ln R_{int}} \right\rceil \qquad ...(18)$$

Expression (18) is the integer part of the number, rounded upward. Now the evaluation of the total number of iterations takes the form:

$$M_{int} = P_{int} R_{int} \qquad ...(19)$$

On the basis of ratios (18), values $R_{int} = 2$ and $R_{int} = 4$, the values $|z_{01}|_{max}$ and specified c_p can be determined P_{int}. At the same time to simplify and universalization of the algorithm (3) it seems appropriate to create multipurpose templates that specify the number of the loops and iterations in the loops. As one of the variants of such a template we shall make the calculation for $|z_{01}|_{max} = 1$, and, for example, $c_P = 2^{-14}$ (any potential difference on linear system the small deviations $z_{r01} \neq 1$ can lead to changes in the number of iterations only in the first loop, which in general may not have a significant impact on the total number of iterations).

For $|z_{01}|_{max} = 1$, $c_P = 2^{-14}$, $R_{int} = 2$ and P_{int} (18) we get on the basis of (19):

$$P_{int} = 14,\ M_{int} = 28 \qquad ...(20)$$

Ad-hoc the iterative process is divided into 14 loops with the following values c_l for each loop:

$$c_l = 2^{-l},\ l = \overline{1,14} \qquad ...(21)$$

Similarly, for $|z_{01}|_{max} = 1$, $c_P = 2^{-14}$, $R_{int} = 4$:

$P_{int} = 7, M_{int} = 28$ ...(22)

and iterative process is divided into 7 loops with the following values c_l for each loop:

$$c_l = 2^{-2l},\ l = \overline{1,7} \qquad ...(23)$$

Based on the above calculations, the duration of the iteration process in the considered conditions is identical at values $R_{int} = 2$ and $R_{int} = 4$. With this approach it is possible to use one of these values as obtained, respectively, of (20), (22) and the calculated weight of the transformation quanta (21) and (23) may be presented as a multipurpose template for the implementation of the iterative process when $c_p = 2^{-14}$. Similarly, we can build templates for other values c_p. With a further increase of the value R_{int} the value M_{int} increases.

In the idealized case described above, the iteration process may reduce the number of iterations in the loop per unit by inserting an additional condition:

$$sign(z_{rRl} - c_l sign(z_{rRl})) = -sign(z_{rRl})$$

or

$$sign(z_{rRl} - c_l sign(z_{rRl})) = 0.$$

This decrease in the number of iterations is provided due to the fact that the analysis of the possible transition through zero is performed using a simple prediction. In this case, the residual can actually continue and pass through zero, but is small in absolute value of the order c_l. In the real computing process, when $\nabla y_{ril} \neq 0$, it is proposed to use the additional terms and condition (6). The iterating in each loop should be executed until across all equations of linear system will execute at least one of these conditions.

Optimized evaluations R_{int} and P_{int} are obtained for the condition $\nabla y_{ril} = 0$. In the real computing process, in accordance with the earlier findings, the number of iterations can be larger or smaller concerning the value R_{int}.

The experimental results

Performance of the received recommendations for the organization of the iterative process has been tested on the solution of various linear systems, characterized by different convergence in the performance of simple iteration method. Following are the results of individual experiments based on linear systems given below on (norm of the matrix coefficient A of examples (26) and (27) greater than one):

$$\begin{cases} y_1 + 0.09y_2 + 0.13y_3 = -0.97; \\ 0.12y_1 + y_2 + 0.11y_3 = -1.13; \\ 0.16y_1 + 0.07y_2 + y_3 = 1.04. \end{cases} \qquad ...(24)$$

$$\begin{cases} y_1 + 0.6y_2 + 0.08y_3 = -0.356; \\ 0.12y_1 + y_2 + 0.7y_3 = -0.604; \\ 0.11y_1 + 0.4y_2 + y_3 = 0.353. \end{cases} \qquad ...(25)$$

$$\begin{cases} y_1 + y_2 + 0.2y_3 = 1.37; \\ -0.8y_1 + y_2 + 0.2y_3 = 0.98; \\ -0.4y_1 + 0.7y_2 + y_3 = 1.13. \end{cases} \qquad ...(26)$$

$$\begin{cases} y_1 + 0.9y_2 + 0.4y_3 = 0.85; \\ -y_1 + y_2 - 0.6y_3 = 0.69; \\ -1.3y_1 - 0.3y_2 + y_3 = 1.25. \end{cases} \qquad ...(27)$$

The comparative analysis between methods of linear systems solutions on the condition of ensuring identical accuracy (~ 2^{-14}) on the basis of the first order delta-modulations with the constant quantum $c=2^{-14}$, by simple iteration method, by the method presented in work (Kravchenko, 1983), and by the method which is also considered in this work on the basis of the first order delta-modulations order with variable quantum at $R_{int} = 2$, $R_{int} = 4$, $R_{int} = 8$, $c_p = 2^{-14}$ was carried out during the research. Conducting the adequate comparative experiments on algorithms presented in this paper (Malinowski *et al*., 1977; Boyong, *et al*., 1977, Tretyakov, 1978, Tretyakov, 1978, 1962; Gomozov 2009; Gomozov and Ladyzhenskii 2010) was not possible in due to the absence of the algorithmic recommendations for the effective completion of the iterative processes in loops (a priori assignment of numerical values of the coefficients used for the completion of iterations with the error, or set the number of iterations to be performed) in these works.

The obtained results are presented in table 1.

DISCUSSION

The analysis of the data in the table shows that the developed algorithm of iterative process of the linear systems solution with the variable quantum, based on the optimized assessments R_{int} = 2, R_{int} = 4, and providing the real time optimization of the iterative processes differs significantly (by the hundreds – thousands times) with the reduction of the number of iterations in relation to the method of the linear systems solution on the basis of the first order delta-modulations, as well as substantial proximity on the number of iterations to the fixed point iteration method and in some cases represents the advantage over the fixed point iteration method (Bakhvalov, 2006; Greenbaum, 1997; Vuik, 2012; Berezin, & Zhidkov, 1966; Faddeev, & Faddeev, 1960). According to the table, the ratio of the number of iteration method for the fixed point iteration method to the method with the variable quantum is as follows ~ 1.7-0.4. At the same time, the table also indicates that when deviated from the optimized values R_{int}, and use R_{int}= 8 the number of iterations will significantly increase.

The data presented in table 1 shows that iterative processes while ensuring the convergence of linear systems may be successfully implemented even in the norm of the matrix of coefficients greater than one.

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CONCLUSION

In this article for the first time it is offered a theoretical substantiation of the selection of the number of loops and the values of the variables quanta, aimed at minimizing the number of iterations in the linear systems solution on the basis of the first order delta-modulations. In addition to the well-known correlation between adjacent quanta ($c_l = 2^{-l}$, $l = \overline{1,N}$) shows the feasibility of using ($c_l = 2^{-2l}$, $l = \overline{1,(N/2)}$), which may be of particular interest when working with a norm of a matrix that is greater than one. The algorithms which showed efficiency on ensuring timely completion of iterations, simple in realization and not demanding any numerical assessment with use of special basic data are introduced for the first time for the formation of the completion moment of the real iterative processes in the loops.

When using the proposed algorithms to build the special-purpose computing facilities the prerequisites are created for substantial reduction of hardware resources due to the ability to exclude the multiplication operator of the multi-digit code and ensuring the performance of the simple iteration method at the level of the iterative processes.

The materials stated in this work shouldn't be considered as the finished recommendations for design of the corresponding specialized computing devices. The architecture of the special-purpose control unit can undoubtedly impact on final algorithmic efficiency, in particular, for example, the possibilities of the organization of simultaneous shifts of coefficients with preservation of shift of the previous loop or without preservation that can be connected with need of essential expenses of the equipment or temporary resources. There is no doubt that the level of the parallelization processes has an impact on the performance of the special-purpose control unit. The ultimate effectiveness is estimated by the aggregate of the assessments of the performance and hardware costs, in this connection the further comprehensive development of the theoretical and applied research is important.

Use of the optimized the second order delta-modulations for the solution of linear systems, and in particular for the effective iterative solution of linear systems with the variable right part of equations can be one of the perspective directions of complex researches of such kind. Basis for carrying out these researches are the scientific results published in the works (Kravchenko, 2008) and developed in the solution of scientific and applied problems in various areas of the science and engineering (parallel data processing in the special-purpose control unit, synthesis of digital control algorithms, compression and protection of signals, splines in the computer graphics). For the first time the author received sustained optimized performance and accuracy of the algorithms of the second order delta-modulations that enables to create in particular the computation process of the iterative linear systems solution with the permanent module on the quanta and by exception of the multidigit multipliers as well as when using the first order delta-modulations. However, increment bit variables are formed that provides essentially different nature of the transitional (iterative) processes, and in case of the constant quantum it allows reducing the number of iterations of linear systems solution by hundreds of times as compared to using the first order delta-modulation. At the same time, the solution of the problem from the point of view of the variable quantum and optimization on the number of iterations, the duration of the iteration and the necessary of the hardware resources, seems along with the use of the first order delta-modulation, must be associated with a phased solution to the theoretical and structural and computational issues.

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